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PROBLEM SOLVING IN ELEMENTARY MATHEMATICS CURRICULUM

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ABSTRACT

The aim of this study is to investigate the effects of Year 6 Elementary Mathematics Curriculum on problem solving. The study had an experimental design and consisted of a total of 120 students in experimental (60) and control (60) groups. The results of students' problem solving performances indicated some prominent findings. First of all, students in the experimental group were more successful at problem solving than the control group students. Second, although neither group achieved a satisfactory success level (≤0.75), the results of students in the experimental group were more homogeneous. Similar results were observed in problem solving stages as well. For all steps of problem solving (understanding the problem, devising a plan, carrying out the plan and looking back at work), the success rates of the students in the experimental group were higher than that of the students in the control group. These results suggested that instead of teaching problem solving as a separate subject, it should be taught as a process interwoven into the whole mathematics instruction where all themes include problem solving activities. Therefore, students' problem solving skills can be improved. Furthermore, there was evidence that the students' ability to use problem solving strategies was enhanced in this way.

Keywords: Problem solving, elementary mathematics curriculum.

INTRODUCTION

As the importance of information increases each day, people's understanding of the concept of "information" also changes. Moreover, with the added influence of the fast advancing technologies, there are changes in the ways and speed of accessing information. Consequently, in order to keep up with all these changes, society's expectations of the skills of the individuals also change (NME, 2006).

While the primary aims of the curricula, previously, were to develop operational skills and to bring up individuals who can perform operations fast and correctly, in the current educational systems such aims are not the priorities of the curriculum. Instead, conceptual understanding has become the principal aim of curricula (NCTM, 2000). Likewise, the renewed primary mathematics curriculum adopted in Turkey reveals a similar understanding with its approach stated as the "conceptual approach" (NME, 2006). The conceptual approach requires more time to be spared for the construction of conceptual foundations of mathematical information and thus establishing relationships between conceptual and operational knowledge and skills (NME, 2006). According to Hiebert and Levefre (1986), when thinking about learning and teaching mathematics for young children, it is useful to distinguish between conceptual knowledge and procedural knowledge (cited in Hiebert and Lindquist, 1999). This is also defined as "relational understanding" (Van de Walle, 2004; Baykul, 2009).

Conceptual knowledge is knowledge that is rich in relationships. It can be thought of as a connected web, where every piece of information is related or connected to other pieces of information. Students acquire conceptual knowledge if they can fit a new piece of information with something they already know or if they suddenly recognize a connection between things that they previously learned as isolated pieces of information. Procedural knowledge, in contrast, is made up mostly of rules, procedures, or algorithms for performing mathematical tasks. Procedures are step-by-step prescriptions that generate correct answers for particular kinds of problems. Both conceptual knowledge and procedural knowledge are important, and both can be learned in school (Hiebert and Lindquist, 1999). Moreover, in such settings, procedural fluency and conceptual understanding can be developed through problem solving, reasoning and argumentation (NCTM, 2000).

Mathematical reasoning offers powerful ways of developing and expressing insights about a wide range of phenomena. People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects. Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understanding (NCTM, 2000). For many students, problem solving means learning the contents of a set of lecture notes and applying this knowledge to specific problems clearly related to the material taught (Tall, 2002). In other words, the problem solving process is a means of finding solutions in unfamiliar situations. And one of the major reasons for studying mathematics is to develop the ability to solve problems. This ability is critical not only to children's future needs and uses of mathematics but also to productive citizenship and even human progress (Worth, 1999).

Due to the critical importance of developing problem solving skills, these skills occupy a substantial part of the curricula. Thus, children enter school with a great deal of informal or intuitive knowledge of mathematics that can serve as the basis for developing understanding of the mathematics of the primary school curriculum. Without formal or direct instruction on specific number facts, algorithms, or procedures, children can construct viable solutions to a variety of problems (Carpenter et al., 1999). Therefore, it is crucial to develop problem solving skills at the primary school. Likewise, problem solving skills, which is the topic of this study, also exists in the primary mathematics curriculum as one of the fundamental objectives of both general skills and field-related skills. Sub-skills that constitute the problem solving skill in the curriculum are: "understanding the problem, identifying sub-stages or the roots of the problem if necessary, devising a plan to solve the problem as appropriate, observing the studies during the procedures, changing strategies and plans if necessary, testing the methods, evaluating the data and information obtained at the solution process, evaluating the significance and relevance of the solution once obtained and detecting new problems" (NME, 2006). Moreover, Polya's (1945) stages of problem solving (understanding the problem, devising a plan, carrying out the plan and looking back at work) are also emphasised.

The curriculum includes a wide range of statements such as not to have an algorithmic and rule based approach to problem solving, to provide opportunities for students to work on problems, to arrange contexts for students to be creative, to emphasise the process and not the outcome, to guide students in using different problem solving strategies, not to provide solutions readily, to ensure appropriate contexts for students to construct their own solutions and to include activities of problem posing (NME, 2006). Although such rich statements in relation to the development of problem solving are presented in the introduction of the curriculum, of the sample activities for about 41 learning areas and for the related inter disciplines, only a few requires the use of problem solving strategies. Similarly, for all three years, problem solving skills do not get enough emphasis in both the gains in the curriculum and the main table of the curriculum, which includes sample activities in relation to these gains. This also contradicts the statements in the introduction of the curriculum.

Given that it is crucial to develop problem solving, an important cognitive skill, during primary school years when cognitive development is fast (Baykul et al., 2010), it is relevant to investigate whether students acquire the skill or not. In particular, this study aims to explore the role of the curriculum in developing problem solving skills of the students as stated in the introduction of the curriculum and the efficiency of different learning contexts designed in relation to the curriculum. In other words, the aim of this study is to identify the efficiency of the renewed primary mathematics curriculum in developing primary school 6th year students' problem solving skills and of the activities designed in line with the mathematics teaching principles as stated in the curriculum.

METHOD

Research design

Experimental research design was used in this study in order to test the activities developed to improve primary school 6th year students' problem solving skills, to observe the development of this skill and to explore the differences between this group and other groups where similar activities were not used.

Participants

120 students registered at the 6th year of two primary schools in Konya, Turkey participated in the study. The schools were similar in terms of both their student and teacher profiles. In both schools all students in Year 6 participated and while students in one school constituted the experimental group, the ones in the other took part in the control group. Prior to the study, any differences between the experimental and control group students in terms of their problem solving skills were identified with a pre-test. The pre-test results indicated that the differences between the experimental and control groups both in terms of their general problem solving scores and their scores of problems solving stages were not significant. This implied that the students belonged to the same population for these qualities and that the groups were equivalent.

Procedures

Following the grouping of the students, activities designed in line with the primary mathematics curriculum were used during a semester in both groups. Again in both groups the primary mathematics curriculum was used as it was. The sequence of the topics and the teaching time of these topics were kept constant in both groups. While teaching in the experimental group the daily plans and activities designed by the researcher were followed, teaching in the control group the activities provided in the teachers' book (NME, 2006a) which was sent free to the teachers by the National Ministry of Education (NME) were followed. This set up provided an opportunity to observe and evaluate the curriculum in terms of problem solving both in a teaching situation suggested by the NME and in a different teaching situation. Throughout the teaching process, tests were administered to both groups at the end of each unit as well as the end of the semester, which provided the basis to the comparisons made in this study. Throughout the study, 3 different evaluations were carried out and problem solving tests that were designed by the researcher were used for these evaluations. The tests were scored out of 100 using a scoring key in order to identify general problem solving performance and using the rubric presented in the appendix in order to identify performance in the problem solving stages. The results were obtained separately for both scores. Students' problem solving strategies that were used in the problem solutions were also identified and elaborated.

Data collection tools

Four different problem solving tests, which were designed by the researcher, were used in the study in order to identify problem solving skills. The first of these was used in identifying students' prior learning and the equivalence of the groups. The other three were the tests used at the end of the units and the semester. The prior learning test consisted of 15 open ended items (9 problem solving, 5 problem posing and 1 matching) which measured fundamental concepts such as the four arithmetical operations with natural numbers, exponential numbers, fractions, arithmetical mean, measuring time and polygons. The reliability study conducted using the scores obtained from the test revealed a Cronbach's alpha (α) internal consistency coefficient of 0.78. The first of the problem solving tests consisted of 15 open ended questions (8 problem solving and 7 routine operational problems) which measured the fundamental concepts of the four arithmetical operations with natural numbers, exponential numbers, lines, planes, line segments, number patterns and rules of divisibility. The second of the problem solving tests consisted of 17 open ended questions (9 problem solving – posing, 5 routine operational problems and 3 gap filling) which measured the fundamental concepts of greatest common factor, smallest common multiple, operations with sets, absolute value, integers and operations with integers. The third test consisted of 23 items (14 problem solving – posing, 6 routine operational problems and 3 gap filling) which measured the fundamental concepts of angle and the sections the angle separates on the plane, complementary and supplementary angles, fractions and their operations, polygons and measuring time. The α reliability coefficients calculated based on the test scores were 0.86, 0.88 and 0.88 respectively.

Data analysis

Data obtained from the problem solving tests were analysed using descriptive statistics such as frequency, percentage, arithmetical mean and standard deviation. Moreover, one-sample t-test was used in order to explore the differences between the mean scores in terms of the learning level accepted as sufficient in this study (0.75; Bloom, 1998). Furthermore, for comparisons between groups, independent samples t-test and F test were used. Finally, the variability coefficient (standard deviation/mean) was used when interpreting group results.

FINDINGS

In what follows, results obtained from the pre-test, results in relation to problem solving skills and finally results in relation to problem solving strategies are presented respectively.

Pre-test results

Prior to the study, students in the experimental and control groups were given a pre-requisite knowledge test. For the experimental and control groups, problem solving means were 44.71 and 40.87 out of 100 respectively. The standard deviations were 16.54 and 17.55 and the variability coefficients were 0.37 and 0.43 respectively. The mean scores of the groups were compared using a t-test, and the standard deviation and variability coefficients were compared using an F test. The results indicated that differences between the groups were not significant at $\alpha=0.05$ level. Likewise, differences between the mean scores for problem solving stages were also not significant at $\alpha=0.05$ level (while mean scores of the stages for the experimental group were 1.50, 0.39, 0.43 and 0.29, that of the control group were 1.46, 0.41, 0.47 and 0.26). This indicated that the two groups were not different – they were equivalent – in terms of a) the measured quality, b) arithmetic means, and c) standard deviations; and thus it suggested that both groups belonged to the same population. The study continued after ensuring equivalence. The results of the problem tests are presented below.

Problem solving test results

Using the scores obtained from the three problem solving tests administered at the end of the units and the semester, students' general problem solving scores were calculated and interpreted. The scores of the experimental and control groups are presented in Table 1.

Table 1: Statistics in relation to problem solving scores (overall scores)

	Experimental Group			Control Group			t and F values
	X	SD	SD/X	X	SD	SD/X	
1st Test	48.98	18.69	0.38	18.55	13.29	0.72	t=10.016** F=1.978**
2nd Test	66.21	20.50	0.31	21.18	16.51	0.78	t=12.907** F=1.542*
3rd Test	33.21	17.30	0.52	13.55	8.37	0.62	t=7.237** F=4.272**

* significant at $\alpha=0.05$; ** significant at $\alpha=0.01$

As shown in Table 1, while the general problem solving scores of the experimental group students were 48.98, 66.21 and 33.21 out of 100; that of the control group were 18.55, 21.18 and 13.55. In other words, the mean scores of all the students, in terms of success percentages, were between 0.14 and 0.66. This implied that students neither in the experimental group nor the control group could achieve the adequate learning threshold (0.75).

On the other hand, score differences of 30.43 in the first test, 45.03 in the second and 19.66 in the third were significant at level α=0.01 in favour of the experimental group. Similarly, when variability coefficients of the groups were compared, the coefficients of all three tests were significantly smaller in the experimental group at level α=0.01 than that in the control group. Thus, in all three tests, the variability of the experimental group was significantly smaller than the variability of the control group.

In short, success at problem solving was higher in the experimental group where learning-teaching activities were appropriate to the learning gains than success at problem solving in the control group. Moreover, students' scores in the experimental group became more homogenous when compared to that of the control group. However, the mastery learning level of 0.75 was not achieved. This indicated that target success levels of the curriculum were not achieved by using neither of the two different activity types (activities in the teachers' book – researcher's activities).

In order to explore students' development in problem solving more clearly, the problem solving tests were scored using the rubric presented in the appendix and each problem solving stage were given a score within the interval 0-2. These scores were compared similar to above. Relevant data are presented in Table 2.

Table 2: Statistics in relation to problem solving scores (scores at problem solving stages)

		Experimental Group			Control Group			t values
		X	SD	SD/X	X	SD	SD/X	
1st Test	Understanding problem	1.00	0.50	0.50	0.32	0.33	1.03	8.585**
	Devising a plan	0.94	0.48	0.51	0.25	0.27	1.08	9.628**
	Carrying out the plan	0.72	0.38	0.53	0.22	0.25	1.14	8.217**
	Looking back	0.29	0.35	1.21	0.02	0.09	4.50	5.544**
2nd Test	Understanding problem	1.85	0.35	0.19	0.89	0.58	0.65	10.513**
	Devising a plan	1.70	0.42	0.25	0.39	0.43	1.10	16.402**
	Carrying out the plan	1.46	0.46	0.32	0.51	0.42	0.82	11.472**
	Looking back	0.74	0.45	0.61	0.05	0.15	3.00	11.196**
3rd Test	Understanding problem	1.32	0.48	0.36	0.76	0.35	0.46	6.578**
	Devising a plan	0.97	0.49	0.51	0.34	0.27	0.79	7.906**
	Carrying out the plan	0.75	0.48	0.64	0.28	0.25	0.89	6.019**
	Looking back	0.45	0.48	1.07	0.04	0.12	3.00	5.932**

** significant at α=0.01

According to the scores given in Table 2; while the mean scores of the experimental group students in relation to problem solving stages were between 0.29 and 1.85; that of the control group were between 0.02 and 0.89. In both groups the lowest means were found in the first test at the looking back at work stage; and the highest means were in the second test at understanding the problem stage. In the experimental group, while students' success percentages in the second test in relation to understanding the problem and devising a plan were 0.93 and 0.85 respectively (score/max. score=percentage; 1.85/2.00=0.93 and 1.70/2.00=0.85), their percentages in relation to the other stages were lower than the 0.75 threshold value. On the other hand, in the control group, the percentages in all tests for all stages were below the 0.75 level. These results suggested that like students' general problem solving scores, their scores of problem solving stages could not reach the target learning levels either. Moreover, in all three tests all the differences between the mean scores of problem solving stages of the experimental and control group students were significant at α=0.01 level in favour of the experimental group students. Hence, in all three tests, in all stages of problem solving, experimental group students achieved significantly higher scores than control group students.

Given that the possible scores based on the rubric were between the interval [0;2]; students in the experimental group understood the problem correctly in the first and second tests; at the stage of devising a plan, they slightly regressed and were at the level of devising the plan correctly or partially correctly; at the stage of carrying out the plan they were at the level of partially solving the problem and at the stage of looking back at work, they couldn't verify or partially verified the problem. When a similar analysis was conducted for the control group, for all three tests, at the stage of understanding the problems the students totally or partially misunderstood the problem; at the stage of devising a plan they couldn't devise an appropriate plan for the solution; at the stage of carrying out the plan they couldn't solve the problem or reached a wrong answer; and at the stage of looking back at work they couldn't verify the answer.

In all three tests, in both the experimental and control groups, for the scores of problem solving stages, a steady decline from the stage of understanding the problem to the stage of looking back at work was observed, and this was expected. Such that, a student who cannot read and understand a problem well cannot be expected to devise an appropriate plan; one who cannot plan cannot be expected to achieve a correct solution; and one who does not have the right solution cannot be expected to do a correct verification. Parallel to the decline observed in the scores of the stages, there was a steady increase in variability coefficients; hence the heterogeneity of the groups gradually increased.

Results of problem solving strategies

Although students' use of problem solving strategies are more related to learning-teaching activities than the curriculum, the influence of strategy use on success at problem solving cannot be ignored. Moreover, the teacher of the curriculum should encourage strategy use and teacher books should facilitate that. With this in mind, experimental and control group students' problem solving strategies were analysed. Therefore, the strategies used by the students in answering the questions in the problem solving tests were identified. Quantitative results in relation to students' use of strategies are presented in Table 3.

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Table 3: The distribution of the experimental and control group students in relation to their frequency of using problem solving strategies

		Experimental Group		Control Group	
		n	ratio	n	ratio
1st Test	No strategy used	20	0.34	42	0.76
	Only one strategy used	30	0.52	13	0.24
	More than one strategy used together	8	0.14	-	-
2nd Test	No strategy used	3	0.05	26	0.48
	Only one strategy used	25	0.42	20	0.37
	More than one strategy used together	31	0.53	8	0.15
3rd Test	No strategy used	16	0.31	24	0.56
	Only one strategy used	17	0.33	15	0.35
	More than one strategy used together	19	0.36	4	0.09

According to the data presented in Table 3; while in all three test in general about 0.30 of the students in the experimental group (0.34, 0.05 and 0.31) did not use any strategies in problem solving, the same ratio rose up to 0.76 in the control group (0.76, 0.48 and 0.56). The ratio of the students who used only one strategy in the experimental group was between 0.33 and 0.52; and those who used two or more strategies were between 0.14 and 0.53. In the control group, these ratios of the students who used only one strategy were between 0.24 and 0.37; and for those who used two or more strategies were between 0.00 and 0.15. Therefore, this implied that experimental group students used more strategies in problem solving than control group students. The ratio of the students who used more than one strategy in the control group was low. Even though strategy use is more related to the learning-teaching activities, this might be interpreted as a sign of inability of the curriculum and the teacher's book to encourage and facilitate the teacher to use strategies.

RESULTS

When learning-teaching activities are designed and carried out in relation to the requirements of the learning gains, a higher rate of problem solving success can be achieved than when teaching is simple-minded. However, in both situations mastery learning level of 0.75 was not achieved. This suggested that the year 6 mathematics curriculum should be reviewed for problem solving.

It is important to emphasise that the variability coefficients of the experimental group were significantly lower than that of the control group. Thus, provision of problem solving skills could be said to have decreased the differences between the students in the class. When interpreted with the mean scores, the learning curve is observed to be skewed to the left and to become sharper, which is a desired outcome. Success could be increased at the stages of understanding the problem, devising a plan and carrying out the plan when training is provided than when it is not. However, the looking back at work stage is problematic. Expected increase was not observed in the group that received training. Several potential reasons could be not attaching importance to the verification of the problem in the school years prior to Year 6, and insufficient training time for this stage, which might have needed longer time.

Students in the experimental group were observed to use problem solving strategies more than the students in the control group. Experimental group students could use one or more strategies together even in the solution

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of routine operational problems, and even more so, they could use original strategies in problem solving. This suggests that when strategies are emphasised in learning-teaching activities, students are able to use these strategies. Therefore, it is possible to conclude that an emphasis on problem solving strategies in learning-teaching activities can increase success at mathematical problem solving.

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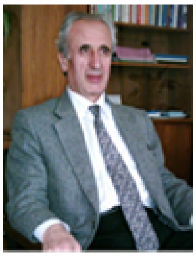

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APPENDIX

Rubric

Problem Solving Stages	Scores	Descriptions
Understanding the problem	0	The problem is totally misunderstood.
	1	Some part of the problem is misunderstood or misinterpreted.
	2	Problem is understood.
Devising a plan	0	Plan is inappropriate to the problem.
	1	Partially correct plan is prepared for the solution.
	2	Correct result could be obtained when the plan is applied properly.
Carrying out the plan	0	Either the answer is wrong or application of the inappropriate plan resulted in wrong answer.
	1	Operational error, wrong answer due to misunderstanding the question, question partially solved.
	2	Correct answer is provided.
Looking back at work	0	The accuracy of the answer is not verified.
	1	The answer is partially verified.
	2	The accuracy of the answer is verified.