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THE PROCESS OF CONSTRUCTING ABSOLUTE VALUE FUNCTION KNOWLEDGE FOR HIGH SCHOOL STUDENTS

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ABSTRACT

In recent years, there have been important changes in the construction of learning environment due to the scientific developments regarding cognitive process. This study aims to analyze high school students' processes regarding construction of the knowledge in the absolute value function in a learning environment prepared according to the recent developments. Using a case study method with a volunteer high school student regarding the teaching part of the project, this study has proposed two problems which expected to give student an opportunity to use his/her pre-experience and knowledge in a maximum level, and these two problems have been used respectively. The study has found out that, in solving the later problem, the student used the knowledge that s/he had acquired from the first two problems and that s/he would be able to conceptualize the absolute value function correctly in a certain level. The study has also found out a certain contribution of using environmental incidents and problems to teaching functions.

Key Words: Abstraction process, Absolute value function, Construction theory.

INTRODUCTION

The process of constructing knowledge in one's mind has always been a subject which attracts the attention of researchers in the field. Even though it is studied to define the suitable learning environment and conditions for this process–since how the knowledge is constructed in the learner's mind is not observed-no consensus has been reached among researchers in this respect. "The Constructivist Learning Theory"- one of the theories effective on teaching mathematics - adopts the notion that the individual constructs the knowledge him/herself. This aspect has directed the researchers to study and explain the process of knowledge construction - that is "abstraction" - in more detail.

The issue of abstraction is an important one in mathematics education, yet there is no consensus upon the abstraction concept (Ohlsson & Regan, 2001; van Oers, 2001). The subject of this study is about the abstraction of the knowledge of the absolute value function. *Abstraction*, in the simplest way is known as "*ascending from concrete to abstract"*. The concept of abstraction has primarily been a subject of interest for epistemologists, then as the studies upon the process of learning have increased, it has also been a field of research for educationists. As for Hershkowitz et al., abstraction is defined as follows;

"Abstraction is as an activity of vertically reorganizing previously constructed mathematics into a new mathematical structure" (Hershkowitz, Schwarz & Dreyfus, 2001).

Abstraction starts from the raw knowledge that the individual has previously constructed, and keeps ready in memory to use in a new activity. There are two major models (empirical and dialectical) of abstraction advanced in the literature. *The empirical abstraction* model starts to take shape by recognizing the similarities

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between different contexts, and results in emerging of a new concept out of these similarities (Mitchelmore & White, 2004). There is an ascending development from concrete to abstract. In view of *the dialectical model*, it is needed to explain the concept of dialectic. The term *dialectic* refers to the definition "thought is in an unceasing motion and change, and the evolution of thought takes place as a result of inner conflicts" (Hershkowitz et al., 2001). In this model of abstraction, a concept is reflected upon, and with each step an even more abstract form of the concept is reached. On the contrary to the empirical model, in the dialectical model there is an advance from abstract to more abstract.

As the *process of abstraction cannot be observed directly* (Dreyfus, 2007), it has been necessary to define observable actions that can give information about the process. In defining the abstraction process, Hershkowitz et al. (2001) has referred to the major observable epistemic actions as *recognizing, building-with* and *construction*, and hence named their model as "RBC". *The RBC Abstraction Model* is based on the activity theory, and comprises three epistemic actions.

Recognizing refers to a familiar structure (Bikner-Ahsbahs, 2004). A previously constructed structure-already used in other situations - is related to this action (Schwarz, Dreyfus, Hadas & Hershkowitz, 2004). Recognizing occurs when the student realizes that the construct that is familiar from a previous activity is connected to or relevant for the mathematical situation in the present activity. It may occur in at least two ways, by analogy and by specialization (Dreyfus, 2007).

Building-with refers to the process of combining familiar pieces of knowledge into a new context. It includes recognizing (Bikner-Ahsbahs, 2004). In other words, building-with is defined as using mathematical structures to achieve a given goal (Schwarz et al., 2004). It reflects recognizing the familiar structure, and using it to solve the new problem. Actually, recognizing and building-with are nested actions where they complete each other. Most often, it is not possible to separate one action from the other.

Construction is the process of structuring new knowledge, also defined as processes of reorganizing and restructuring. Constructing is the process of restructuring and reorganizing what is recognized and known to construct a new meaning (Bikner-Ahsbahs, 2004). According to Ohlsson and Lehtinen (1997), the process of construction - as the central epistemic action of abstraction comprises vertical reorganized knowledge, and requires theoretical thinking. Constructing is observed when the individual uses the structures he recognizes in solving the problem, given to teach a new mathematical concept.

The most significant difference between building-with and construction is that in the former action, existing constructs are used to solve a problem or explain a situation, and in the latter one, a new mathematical concept (or generalization) is constructed. Realizing a new aspect of an existing mathematical concept is also interpreted as construction. If the students solve a standard problem, they are likely to recognize and build-with previously acquired structures. If they solve a non-standard problem, they might get in the process of construction (Hershkowitz et al., 2001).

According to the above mentioned explanations, the epistemic actions are not independent from each other. There are various relations in between the three actions. As discussing these relations in detail would lead to a more through study over the abstraction process, it is needed to include them hereunto.

Recognizing and building-with are often nested within constructing actions. Recognizing can take part in the other two actions. Regarding the three epistemic actions underlying a student's behavior, the action of constructing does not merely follow recognition and building-with in a linear fashion and yet they have a nesting composition. For example, a student cannot get to the building-with and construction stages if he cannot "recognize". Also, a student who can "recognize" has to perform both actions of recognizing and building-with in order to "construct". This mechanism is called *the dynamic nesting of the epistemic actions* (Hershkowitz et al., 2001).

There have been many studies made upon the abstraction process. Some of these studies are about defining the abstraction process, some are about the factors that are effective on the process, and some are about the epistemic actions. In order to analyze the abstraction process, Hershkowitz et al. (2001) have carried out a study with a 9th grade student, concluding that abstraction has occured during problem solving, and that the student has solved some of the problems by recognizing and building-with, and that for some particular problems has needed help from the interviewer to implement construction.

In the study on absolute value functions $(y = |f(x)|)$, Özmantar and Monaghan (2007) have examined the abstraction process in an enviroment where it was possible to communicate with friends, and guidance from the tutor was obtained. In this study, they have put forth the necessity of the four important factors in the abstraction process. These are: *(i) Mediation through man and material, (ii) Guidance of the tutor for mathematical interpretation, (iii) The suitable dialectical environment on student development* and *(iv) The presence of a thing to be abstracted.*

Regarding the factors effecting the abstraction process, Monaghan and Özmantar (2006) have examined the process of constructing one of the factors by utilizing the other over the functions of $y = f(x),\ \ y = f(|x|)$, $y = |f(x)|$ and $y = |f(|x|)|$.

Altun and Yılmaz (2008) have studied the abstraction process of "Greatest Integer Functions" in the light of RBC model with a case study of two junior high school students. The major aim of the study has been to design and discuss a learning environment for high school students to enhance the quality of Greatest Integer Function teaching. With this study, Altun and Yılmaz have shown that students can abstract the knowledge of greatest integer functions, and that the previously constructed knowledge of piecewise functions has provided a suitable basis throughout the process.

Functions play an important role in high school mathematics. In the curriculum, 24 hours have been allocated for the concepts of relations, functions and operations in 9th grade, 22 hours for the exponential and logarithmic functions in 11th grade, and 16 hours for the functions, piecewise functions and absolute value functions in 12th grade (Ministry of Education, 2008). This study focuses on *absolute value functions*.

There is no particular purpose in choosing the *Absolute Value Functions* for the study topic. Our primary objective is to create a learning environment where the students can construct meaningful mathematical knowledge, and to apply the designated teaching. It is also our aim to discover clues to enhance the quality of teaching through the process later on by reporting the teaching. The study is conducted to teach absolute value functions at an environment created in regard to the three above mentioned main actions.

The study has been carried out taking into account the perception of mathematics defined as the knowledge and skills acquired during problem-solving and interpretation processes based on modelling of reality (De Corte, 2004).

METHOD

Research Model

This research is a "case study" and in this aspect a qualitative one**.** A case study is an empirical research method which studies a phenomenon within its real life framework, in which boundaries between the fact and the content are not clear, and which is used when more than one evidence or source of data is available (Yıldırım & Şimşek, 2006).

In case studies and qualitative designs, the researcher does not only observe the subject research as in quantitative designs, but participates in the study in person to analyze both the subject and the participants.

The researcher interviews the participants one by one, thus he/she is a part of the process. In this study, the researcher has also undertaken the role of teaching, and therefore has been a *participant observer*.

Study Group

The study has been carried out with a student in the first grade of a high school who participated voluntarily. The mathematical achievement of the student has not been tested, but the school manager and his teachers have been interviewed with about the study. Having a 5 out of 5 as the final semester grade, the student has been reported to be successful at mathematics. The student has not been taught about Absolute Value Functions either at school or any other institution. His knowledge about functions has been consisted of domains, ranges, matching, real number pairs, the coordinate system, and matching the real pairs with the end points on the plane which have all been covered in the 8th and 9th grade curricula.

Data Collection

Qualitative data collecting methods such as observation, interview and document analysis have been used altogether, supporting "*data diversification*". Yıldırım and Şimşek (2006) cite that the basic principle in diversification is collecting data from different individuals and different environments with different methods in order to prevent the prejudices or misunderstandings later on during conluding the research (Koçbeker & Saban, 2005). A video camera and a computer have been used in data collection and analysis.

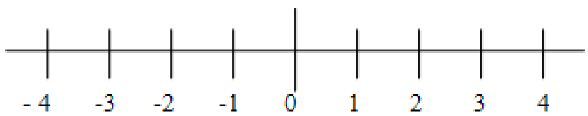
Audio and video recordings throughout the study have been made within the knowledge and permission of the student. At the beginning of the study, questions and explanations have been presented to acquaint the student with the context given in the problems. During the solution stage, new questions have been asked whenever needed to reveal the student's opinions. The student's verbal and nonverbal communication with the researcher have been observed through the study. Later on, all recordings have been analyzed in terms of constructivist learning principles and abstraction process stages.

Data Collection Tools

The data collecting tools used in the case study are the two worksheets containing the problems and function graphs regarding Absolute Value Function. In order to get the expected results out of the clinical interviews in selecting problems, it is required that the problems are ; *(i) discussible, (ii) open-ended, (iii) performing in the way that the student's mental abilities are revealed* (Tanışlı, 2008). The problems used in the study respectively are as follows:

For each problem, the coordinate system is already drawn with the axes designated.

- ***Tank Problem***: The target for a recently developed tank which is tried during a military drill has been shown as a vertical line. During the shooting, if the tank makes a hit either to the right or to the left of the vertical line, the shot is accepted as a miss and the distance between the area hit and the vertical line determines the amount of error. The results of 5 shots are as below. They are +4.3, -2.6, 0, +3.8, -0.9. Mark these values on the graphic below. In case the number of shots increases considerably, design the shape of the graph accordingly and show it by drawing.



- ***Weighing Scale Problem***: While teaching the measures, the teacher brings a weighing scale to the classroom and asks each student to guess the weight of their classmates. Afterwards, every student is weighed on the scale. Students find out the amount of mistake by deducting real weights from their estimates. For example, if the difference is -2 kg, then the amount of error is +2; if it is +2 kg, then the amount of error is also +2. They are trying to find the student who is least mistaken.

— In this case, according to the table below which shows Nisa's and Efe's estimates on three people (A, B, C), is it Nisa or Efe who is most mistaken?

	A	B	C
Real Weights	60	65	50
Efe's Estimation	63	66	45
Nisa's Estimation	61	60	52

- Explain how the margin of error (margin of mistake) has been calculated.
- Think that a lot of people's weights could be predicted. On the coordinate system, draw a graph for each of these people that show the amount of error, so that all kinds of amounts of mistake can be found out from the graph.

The questions prepared in accordance with the explanations given in "introduction", related to constructivist learning model should; (*i*) Keep the students busy with problem solving, (*ii*) Activate the students' pre constructed knowledge and experience as much as possible, (*iii*) Comprise the presence of a thing(s) to be abstracted.

Each problem-printed on A4 paper-has been given to the student one by one in the above order. The second problem is presented to the student only when the first one is solved. The problems chosen for the study qualify to provide a basis to understand the Absolute Value Function.

Analysis of Data

Data have been evaluated by descriptive analysis method. In this type of analysis, data obtained are summarized and interpreted according to the themes previously specified. In descriptive analysis, it is often seen that quotations are included in the evaluation to reflect the opinions of the observed or interviewed individuals intensely. The aim is to present the findings acquired in an edited and interpreted form to the audience (Yıldırım & Şimşek, 2006).

As the study adopts the RBC model framework, first, the audio and image recordings, accompanied with the worksheets have been converted to text in order to observe the epistemic actions of recognizing, building-with and construction. Secondly, the observation notes taken by the researchers during the exercises have been evaluated. Finally, comments have been made based on data to interpret the findings, explain the relations in between, and draw conclusions.

Validity and Reliability of the Study

The concepts of "validity" and "reliability" are essential and traditionally accepted in quantitative research. In qualitative research, it is suggested to use the concepts "credibility or trustworthiness" instead of "internal validity", "transferability" instead of "external validity", "consistency" instead of "internal reliability" and "confirmability" instead of "external reliability" (Yıldırım & Şimşek, 2006). In the qualitative research, validity infers to observing the studied phenomenon objectively and as it is (Kirk & Miller, 1986).

In this study, internal validity is provided by the long term interaction during the research, deep focus data collection, diversification, expert survey and participants confirmation. External validity is provided by description in detail and purposeful sampling. To provide the reliability of the study, the recordings and the observation notes have been surveyed, and interpreted by 2 different experts in terms of observability of the epistemic actions. It has been seen that the interpretations are consistent with each other.

FINDINGS AND COMMENTS

Can's (the student) process of constructing knowledge of "Absolute Value Function" is introduced below considering the epistemic actions of *recognizing, building-with* and *construction*. (C: Can (This is not the real name of the student), A: Researcher).

Can spent 10.12 and 10.32 minutes on the first and second problems respectively.

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Analysis of the Process

Before the first worksheet containing the military drill problem was given to the student, the researcher asked questions to check the student's pre constructed knowledge. The dialogue is as follows;

100A: Can, do you know what the analytical plane is?
101C: Analytical plane. Hmm, that is, two axes.
102A: Or the coordinate system.
103C: I know the coordinate system.

Referring to the student's answers above, it is seen that the student knows the coordinate system, and *recognizes* the knowledge related to this system. Afterwards, the worksheet was given to the student. He was asked to read the question carefully. The student thought upon the problem for a while to understand it. The student could not understand what the target line is and therefore started a dialogue with the researcher.

107C: Target line?
108A: For the tank, now the target is identified as a vertical line (The researcher shows the vertical line). This target will be hit (Points the target line). Here will be hit but not all missiles strike.
111C: They will not.

As seen from the dialogues the student apprehended the target line, and not to be mistaken, marked the target line distinctly on the sketch (Figure 1).

113C: It is shown as… During the shooting, if the tank makes a hit either to the right or to the left of the vertical line, the shot is accepted as a miss.
114A: Show me an example.
115C: For example I can hit there. I hit here. It is a miss.
118A: How much?
119C: Here (Points at the distance to the vertical axis), the distance is the amount of error. Distance to target is 0.4.

The student made a hit on the coordinate system he previously recognized using the knowledge of plotting a point, and told the amount of error corresponding to the hit. Using the expression "distance to target" shows that he *built-with* the recognized knowledge referred in the previous dialogue. The researcher asked the student to make the other sample hits as well as the hits he will set in order to find the amount of errors.

125C: For example here (Marks correctly) (Figure 1).
126A: Yes, in this hit, what is the amount of error?
127C: -2.5
128A: Now will you say -2.5 to it?
129C: As it says the distance between this line and the point hit determines the amount of error, it should be 2.5, not -2.5.
130A: Mark the other hits.
131C: Here is zero (Marks correctly) (Figure 1).
133C: +3.8 falls here
135C: -0.9 is somewhere here

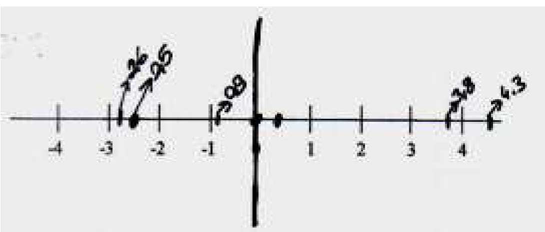

Figure 1: The Graph Can Marked the Hit Amounts

The markings that Can made correctly on Figure 1 in respect to the problem, have shown that he recognized the pre-constructs related to the problem, and that he understood the problem as well.

137C: (Returns to the question) We said "Mark these values on the graph", "In case the number of shots increases considerably, design the shape of the graph accordingly and show it by drawing".

139C: The point hit.

142A: Minus, when hit +4.3?

143C: I will mark here.

144A: How much is the amount of error then?

145C: 4.3. I will mark there. I mean here.

151C: -2.6 stays here. The amount of error of this will be again positive because it says distance. And it stays here.

The markings that the student made correctly according to the researcher's request have shown that the student *built-with* the previously recognized knowledge. In other words, the student knows how to mark ordered pairs in a plane, and that he uses this knowledge to *built-with*. It has been seen that the student noticed that the amount of error is positive when he says "*…The amount of error of this will be again positive because it says distance. And it stays here* (151C)".

155C: It says "In case the number of shots increases considerably, design the shape of the graph accordingly and show it by drawing."

156A: What would happen if it increased?

157C: 1.7 stays on this. If we think of the amount of error like that again, 1.7 and -3.2. And that is here.

158A: If it hits again and again?

159C: Again and again? Uhm, when the number of hits increase a lot… It will not change… I mean I saw it will not change, that is it goes directly proportional all the time.

In this part of the dialogue, it has been observed that the student began to realize how the graph will be shaped. Using the expression "*it goes directly proportional"* in line 159C shows that he built-with previously recognized knowledge into new knowledge constructs. Later on, the student joined the marked points by a ruler, and continued the dialogue as follows;

166A: Yes. What can you say for the end points?

167C: They go infinite.

170A: Does it look like any graph that you recognize?

173C: No.

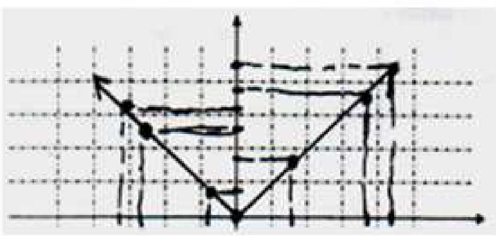

Figure 2: The Graph of "Tank" Question

The student sketched the graph correctly, and replied the researcher's question regarding the end points of the lines as "*go infinite*" (167C). When the researcher asked if he recognized this graph before, he said that he did not. Solving this problem, the student *constructed* a new knowledge that he never came across before or had any opinion about.

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In order to consolidate the constructed knowledge of absolute value function, the second question was given to the student. For the student to comprehend the problem better, the researcher gave examples about himself, and the below mentioned dialogue took place.

203A: Now, how much do I weigh? What do you think?
204C: Hmm, 75 kg.
205A: Yes, I am 73 kg. You' ve been mistaken by 2 kg on my weight. The difference between is called 'the amount of mistake'. For example, I guess you weigh 62 kg. How much do you weigh?
210C: 65.
211A: Yes, I've been mistaken by 3kg. You estimated my weight more and I estimated yours less. The amount of mistake is a value related to the distance to the actual value.

As seen from the dialogues above, it has been observed that the student apprehended the problem better. Later, the researcher asked the student to find the amounts of mistake for the given table. The student found out who was mistaken more by looking at the table.

216C: A weighs 60, C weighs 50.
220C: There…(Points out the correct place) Yes then A, Efe is mistaken by +3, Nisa by +1, we skip to B. Efe is mistaken by +1, Nisa said 60, it is - 5 but we accept -5 as + . We write down +5.
222C: Efe again by +5, Nisa is mistaken by +2 here. If we add these, we will find how much they are mistaken.
224C: Efe is mistaken as +9 in total, and Nisa is mistaken as +8. This shows up Efe is mistaken more.

	A	B	C	
Real Weights	60	65	50	
Estimation of Efe	63 +3	66 +1	45 +5	+9
Estimation of Nisa	61 +1	60 +5	52 +2	+8

Figure 3: The Finding of Amount of Mistake

The student calculated which student was mistaken more by using the information on the table correctly. Figure 3 shows the student's operations while finding out the amounts of mistake. It has also been seen that the student constructed the knowledge that amount of error will be positive when he says *"Nisa said 60, it is -5 but we accept -5 as + "* (220C).

226C: It says "*All kinds of amounts of mistake can be found out from the graph*"*. We will draw such a graph that from this graph all kinds of amounts of mistake can be understood.*
227A: Efe guessed 63 as 60. Where do you think this corresponds onto the graph?
228C: Amount of mistake is 3, amount of error is +3. It corresponds here. The joint of these (Points the correct place in the coordinate system).
234C: +1, +3. Here for example it is normally -5 but because it says +5 to -5, it is there.
236C: It falls somewhere here (Points the correct place in the coordinate system). In this situation the amount of error is again +5.

Giving the amount of error correctly (228C) again – that is the amount of error cannot be negative-the student proves that the constructing action has occured. After evaluating Efe's guesses, the student continued with Nisa's, completing them all correctly. Noticing that no data came up to correspond to origin, the researcher asked a new question to the student to draw his attention on this point.

239A: Alright, how much do I weigh? What do you think?
240C: 80.

241A: You guessed right. How will you mark it?
244C: Like this… on zero (Shows the origin that is the right point).

The student has marked the origin without any hesitation. This has led to the opinion that the student had pre-constructed knowledge about the origin.

245A: In your opinion, how will the graph of these values do?
246C: The graph of these…(Thinking).
247A: Probably it will be useful to write down a few more examples, right?
248C: Yes, right there.
249A: Let's suppose I was 73 kg, and you guessed 75 kg. Where does it correspond to?
252C: It corresponds to 2 to 2.
253A: 2 to 2. OK, I guessed your weight?
254C: I was 66 and you said 62.The difference corresponds to -4.The amount of error to +4. And that falls to there.
256C: A graph in this shape. As it is understood, it is going to be something like decreasing like this and increasing like this. Let's draw that, too. Yes, and this one is in this shape (Implying the graph in the previous question).

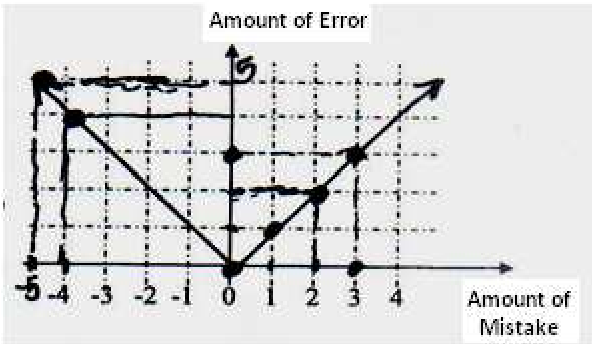

Figure 4: The Graph of "Weighing Scale" Question

The student correctly sketched the graph of amount of error with given amounts of mistake (Figure 4). A few more examples were given as the already given examples were insufficient to draw the graph. It is seen from the statements in 256C that the student knows there are decreases and increases in graphs, and that he uses this knowledge while drawing the graph. The student realized that the graphs he drew in both problems resembled each other a lot. The researcher continued the interview to make the student think upon both graphs.

261A: What is common between these graphs?
262C: Here if distance is concerned in a case, the function is always increasing properly and decreasing properly. Yes this part is y=x (Showing the part in the first zone).
263A: Yes, Where does y=x correspond to?
264C: There (Points correctly).
265A: We can't say the same thing to the part in the second zone, can we?
266C: No we can't. What we will say here is y=-x.
267A: y =-x, here. See, this function is called absolute value function. It is shown as y = $|x|$. *If we write down the two situations together* $y = |x| = \begin{cases} x, x \geq 0 i\c{c}in \\ -x, x < 0 i\c{c}in \end{cases}$ *does this equation represent the graph?*
272C: Yes it does.

Line 262C has shown that the student recognized straight line graphs, increasing and decreasing functions and/or the concepts. Practicing on all of the data, knowledge of absolute value has been constructed together with the researcher. Moreoever, in order to confirm that the knowledge is constructed, the researcher continued the dialogue with the student, talking over the definiton written.

273A: Why do you say it represents?
274C: Because if x<0, –x is correct, it comes out of the absolute value as –x .
275A: If x<0 why do we write down "-" in front of it?
276C: It is negative. It will be positive. To come out as positive. That. Being negative.
278A: Absolute value function is such a function. Have you ever seen an absolute value function?
279C: No I haven't. This is more like… I understood it from the previous question… That distance can never be negative.
280A: Yes, to express such case, such functions are being used.
281C: Good. I mean, it settles in my head better with this. When I see something like this, it can come directly to my mind.

It is seen with this second exercise that the study subject has been consolidated in the student's mind.

DISCUSSION AND CONCLUSION

The main objective of this study was to suggest, implement and discuss an education model that would serve the purpose of teaching Absolute Value Function. To this end, a student who has not yet met the concept of Absolute Value Function has been taught by addressing two related questions in order. Afterwards, it has been examined as to see whether the abstraction process of the knowledge of Absolute Value Function has occured during the teaching.

The questions used in the teaching–together with some limitations have been suitable and sufficient to construct the knowledge of Absolute Value Function by the student. The study can be evaluated in respect to the aim and the aspects taken into consideration during planning as such:

The Compatibility of the Teaching with the Constructivist Theory

In this study, teaching has been based on problem solving where the student has shown interest for both problems and made effort to get results. Eventhough the student has not attended any military drill, he is familiar with the terms "military drill-tank-target " that he hears in daily life. By this means, the problem is qualified to be evaluated as a real life problem.

Yet this situation does not fully coincide with the term "modelling of reality" that De Corte (2004) uses in defining mathematics. For modelling reality, the research should be based upon mathematization (Gravemeijer, 1990). Together with this, the student should be experiencing some difficulties in solving the problem, and then by working upon the problem should be defining the absolute value function and drawing its graph.

Nevertheless, we can say that a constructivist learning environment has developed because the study was student-focused, the teacher guided and allowed the student to "think", and created opportunities for the student to construct the knowledge on his own in the end.

The second problem (weighing scale question) has been addressed with the intention of letting the knowledge structures built via the first problem to be used, and to remove the brittle structures. The situation in the second problem could be created in the classroom. Instead of given estimations of Efe and Nisa, students in the classroom could be weighed, and that would probably be a better case. Other researchers may try this option. Even as it is, the characteristics of constructive learning prevail the study.

The Realization of Abstraction

It is understood in this study that the concept of Absolute Value Function was abstracted in a certain point. The correct and incorrect applications of the behaviours like recognizing, building with and constructing (Dreyfus,

2007) which are defined as observable epistemological actions of abstraction process were observed and reported separately in both problems. In the first problem the students abstracted although in a fragile manner absolute value function by using his prior information, for instance coordinate system. The second problem was asked for supporting the first problem and for removing the fragile parts formed in the first problem and the difficulties experienced in the first problem were not seen in the second problem. The expression of the student such as 'as one can understand, it will be something that increase in this way and decrease in this way (256C)' showed this explicitly. This showed us that student used the new structure in the form of absolute value function which was constructed in the first problem.

It was understood that the concept of absolute value function was abstracted according to the definition which was provided for abstraction by cognitive psychologists (Mitchelmore & White, 2004) as 'relating mathematical objects according to their features and obtaining a more advanced mathematical object'. The explanations regarding the abstraction process came out persistent with the studies of Herskowitz and others (2001) who supported dialectic approach; Monaghan and Özmantar (2006), Dreyfus (2007) and Yeşildere and Türnüklü (2008) and the dialectic nature of abstraction was verified in this study. The non linear structure of recognition, build with and construction which are known as the epistemological actions of abstraction and nested within each other (Dreyfus, 2007; Yeşildere & Türnüklü, 2008) was corrected in this study.

This study points out that using authentic sitiuations or real events in teaching can increase the quality of education. Similar studies carried out on functions lay out the need for reordering the subject order in existing high school programs as linear, quadratic, trigonometric, periodic, exponentiali logaritmic, sign, full value, partial functions (Ministry of Education, 2008) according to order which requires to get configured on each other.

Another result of this study lays out the conribution of real and extra ordinary problems to construction of more quality mathematical knowledge. It is seen that while organizing teaching, chosing appropriate problems is an important factor for on the result. Instead of starting teaching by giving a definition as in traditioanal teaching, studying on real events and problems and making them mathematical make abstraction easy. This kind of a study cause to strength structures acquired before by causing to use them constantly. In addition to all of these advantages, since the problems examplified in this study decided according to the thinking styles of the students, genearlly compatible with group work rather than classroom applications and there are some difficulties in implemeting this to the general. In this aspect of the study, it is though that there is an urgent need to carry out similar studies for teaching with bigger groups.

IJONTE's Note: This study is presented as an oral presentation on 18th Congress of Educational Sciences.

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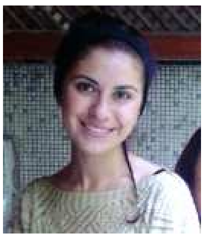

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REFERENCES

Altun, M. ve Yılmaz, A. (2008). Lise öğrencilerinin tam değer fonksiyonu bilgisini oluşturma süreci. *Ankara Üniversitesi Eğitim Bilimleri Fakültesi Dergisi*, 41(2), 237-271.

Bikner–Ahsbahs, A. (2004). Towards the emergence of constructing mathematical meanings. *In Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2, 119-126.

De Corte, E. (2004). Mainstreams and perspectives in research on learning mathematics from instruction. *Applied Psychology*, 53, 279-310.

Dreyfus, T. (2007). Processes of abstraction in context the nested epistemic actions model. EBSCO: http://escalate.org.il/construction_knowledge/papers/dreyfus.pdf

Gravemeijer, K. (1990). Context problems and realistic mathematic instruciton. Gravemeijer, K., Hauvel, M. V., & Streefland, L. (Ed.). *Contexts Free Productions Tests and Geometry in Realistic Mathematics Education*, the State University of Utracht, Neterlands.

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in contexts: Epistemic actions. *Journal for Research in Mathematics Education*, 32(2), 195-222.

Kirk, J., & Miller, M. L. (1986). *Reliability and validity in qualitative research.* Beverly Hills, CA: SAGE Publications.

Koçbeker, B. N. ve Saban, A. (2005). Otistik bir çocuğun yabancı dil öğrenimine ilişkin örnek olay incelemesi. *Çukurova Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, 14, 401-428. Ulakbim: http://www.sosyalbil.selcuk.edu.tr/sos_mak/makaleler%5CAhmet%20SABAN%20Beyhan%20Nazl%C4%B1%20KO%C3%87BEKER%5C401-428.pdf

MEB. (2008). http://ogm.meb.gov.tr

Mitchelmore, M. C., & White, P. (2004). *Teaching mathematical concepts: Instruction for abstraction*. Presented at the 10th International Congress on Mathematical Education, Copenhagen, Denmark.

Monaghan, J., & Özmantar, M. F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics*. EBSCO: http://www.springerlink.com/content/c134370723467362/fulltext.pdf

Ohlsson, S., & Lehtinen, E. (1997). Abstraction and the acquisition of complex ideas. *International Journal of Educational Research*, 27, 37-48.

Ohlsson, S., & Regan, S. (2001). A Function for abstract ideas in conceptual discovery and lerning. *Cognitive Science Quarterly*, 1(3), 243-277.

Özmantar, M. F., & Monaghan, J. (2007). A dialectical approach to the formation of mathematical abstractions. *Mathematics Education Research Journal*, 19(2), 89–112.

Schwarz, B., Dreyfus, T., Hadas, N., & Hershkowitz, R. (2004). Teacher guidance of knowledge construction. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 4, 169-176.

Tanışlı, D. (2008). *İlköğretim beşinci sınıf öğrencilerinin örüntülere ilişkin anlama ve kavrama biçimlerinin belirlenmesi.* Yayınlanmamış doktora tezi, Anadolu Üniversitesi Eğitim Bilimleri Enstitüsü.

van Oers, B. (2001). Contextualisation for abstraction. *Cognitive Science Quartely*, 1(3), 279-305.

Yeşildere, S. ve Türnüklü, E. B. (2008). İlköğretim sekizinci sınıf öğrencilerinin bilgi oluşturma süreçlerinin matematiksel güçlerine göre incelenmesi. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi*, 22(1), 485-510.

Yıldırım, A. ve Şimşek, H. (2006). *Sosyal bilimlerde nitel araştırma yöntemleri*. Ankara: Seçkin Yayıncılık.