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PROOF STARTEGIES PREFERRED BY STUDENTS IN THE GEOMETRY

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ABSTRACT

New geometry program in Turkey was developed by TTKB and became effective in 2009. The most significant innovation contained in the program its use of synthetic, vectoral and analytical approaches simultaneously towards proof in geometry. In addition the program encourages geometry teachers to make common the use of all these approaches among students instead of conventional synthetic approach. The first graduates of this program completed their education 2013. Analysing the use of approaches by these students in solving problems is siginificant in that it may provide useful insights about the efficiency of the program. The study aims at identifying proof strategies preferred by students and the reasons for their choices. For his aim 51 senior high school students attending three different Anatolian High Schools participated in the study. They were asked to answer two questions which can be solved through using all three approaches towards proof. They were also asked to indicate the reason for their choice of the approach they used. The findings of the study showed that majority of the students used the synthetic approach while solving the problems.

Key Words: Geometry program, proof, approaches towards prof.

INTRODUCTION

Geometry is part mathematics and helps children to develop relationships between geometrical patterns in universe and other fields of methematics. In addition through geometrical knowledge children could successfully deal with problem-solving and other solution-required daily routines and school subjects. Geometry education should start in basic education. However, given that in geometry courses at the level of secondary education students experience significant difficulties the course at the level of basic education is not effective. Geometry as a field of study has developed further than it was expected due to its scope and differential sub-branches. Therefore, educational planners come across such questions as which of these advances and concepts in geometry should be included in educational programs from pre-school to universty levels? and Which geometrical topics and concepts are much proper to be taught (MEB, 2011).

Geometry is “a complex interconnected network of concepts, ways of reasoning, and representation systems that is used to conceptualize and analyze physical and imaged spatial environments” (Battista, 2007). Activities related to proof are significant in that they reflect the ability to reasoning. In geometry all types of proof

strategies could be employed (Zaimoğlu, 2012). Sir Christopher Zeeman stated that the scope of geometry include "remembering theorems, understanding proofs, making predictions, seeing truthand employing visual intution" (Royal Society/JMC, 2001). Proofs are significant part of mathematics (Padula, 2006). In mathematics many procedures depend on previous patterns and structures. New and acceptable patterns in mathematics are produced through proofs and other related procedures. Therefore, proofs are necessary basic patterns for mathematics to develop and expand (Mingus and Grassl, 1999). Through proofs problems are shown as either correct or incorrect (Tall and Mejia-Ramos, 2006). In addition to this function proofs also provide the reasons for why something is correct or incorrect (Hanna, 2000). There are numerous studies about the views and attitudes towards proofs among students, student teachers and teachers (Harel and Sowder, 1998; Almeida, 2000; Moralı, Uğurel, Türnüklü and Yeşildere 2006; Coşkun, 2009; Arslan and Yıldız, 2010; İpek, 2010; İskenderoğlu and Baki, 2011; Güler and Dikici, 2012; Köğce, 2012).

Countries attempt to modify educational systems which is regarded as a significant activity. Educational systems should prepare individuals how to uptade their knowledge-base and how to employ different study environments. Therefore, modifications in educational systems are improtant ways to adapt the developments and advances in changes occurred in wider settings (Wedell, 2009). This is necessary to produce those individuals who can positively react to novice environments, adapt to new skills at different periods of their life and contribute to the society where they live. Thus educational changes are significant to achieve such goals. Most of the governments allocate major part of their budget for educational activities. Changes in educational systems are among those regarded by governments as important (Kennedy, 1996). In Turkey revisions of educational programs at the levels of basic education and secondary education have been going on (Kurt and Yıldırım, 2010).

Similarly geometry program in Turkey has been revised and modified. The new geometry program was developed by the board of education in 2009. In the same year it was used for ninth grade. In 2010 it was employed for tenth and eleventh grades. In 2012 the program was began to be used for twelfth grade. A number of new activities and strategies was introduced to geometry course through the new educational program. One of the major novice approaches is to emphasize the significant place of proof in mathematics and geometry. In addition, the program encourages the use of three proof approaches simultaneously. These approaches towrds emphasized by the program are synthetic, vectoral and analytical. More specifically instead of using only synthetic approach towards proofs teachers are expected to use and make the students familiar with all three proof approaches. Based on this change in regard to the proof approaches the aims of the geometry course were expanded to contain the followings: "Students can deal with geometrical concepts using the approaches of synthetic, vectoral or analytical.'' (MEB, 2010) and "Students can recognise the differences among the approaches of synthetic, vectoral or analytical and use them properly. They should use these proof approaches based on their easiness and appropriateness.'' (MEB, 2012).

Geometrical approaches towards prof

Major appraoches towards proof in geometry are briefly defined as follows:

Synthetic approach towards proofs is one which employs postulates. Vectoral approach towards proofs uses vectoral algebra. Analytical approach towards proofs, on the other hand, employs coordinate system (MEB, 2012).

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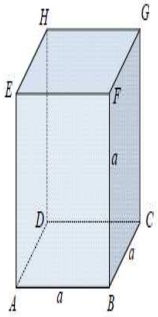
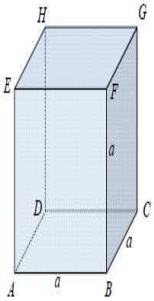
Let's show volume correlation of a cube using three approaches towards proofs:

Synthetic Approach



$V = a^3$

Vectoral Approach

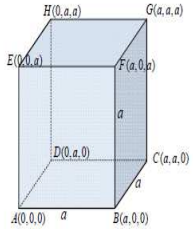


$V = \left\|\overrightarrow{AB} \times \overrightarrow{AD}\right\| \cdot a$

$V = a^2 \cdot a$

$V = a^3$

Analytical Approach



$\{A, \overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{AE}\}$

dik koordinat sistemi seçilerek

$V = \left\|\overrightarrow{AB} \times \overrightarrow{AD}\right\| \cdot a$

$\overrightarrow{AB} \times \overrightarrow{AD} = (0,0,a^2)$

$\left\|\overrightarrow{AB} \times \overrightarrow{AD}\right\| = a^2$

$V = a^2 \cdot a$

$V = a^3$

In order to assess the effectiveness of the new geometry program it is important to reveal which proof approach is used by the students who were given geometry education through this program. The first graduates who were educated by this new program finished school in 2013. The aim of the study is to identify the proof approaches preferred by these students. In line with the aims the study attempts to respond the following research questions:

- Which proof approach was preferred by the 12. grade students?
- What are the reasons for their preferrence over these proof approaches?

METHOD

The design of the study is case study which is among qualitative research techniques. Case study is a research technique which analyses any fact or event in its real setting using multidimensional, systematical and in-depth examinations (Yıldırım and Şimşek, 2005). Case studies are especially appropriate for the individual studies. It provides the opportunity to study a dimension of a research question in depth and in a short time. It also prvides the researcher to focus on a specific topic (Çepni, 2009).

Information about participants, data collection tools, data collection process and data analysis are given in this section.

Participants

The participants of the study were 51 senior high school students attending three Anatolian High School in a province of Turkey.

Data Collection Tools and Procedure

In order to collect the data of the study the participants were given two different geometrical statements which can be proved by three approaches towards proof, namely the approaches of synthetic, vectoral or analytical. The proofs of the statements were also given to the students. The related geometry statements are as follows:

Statement 1*: "The lenght of the line which passes through the middle points of a triangle's sides and is parallel to its base equals to the base lenght of the triangle."*

Statement 2*: "The area of a rhombus equals to the half of its diagonals multiplied."*

In addition the participants were asked to indicate which proof approach they preferred to use and the reasons for their preferrence over the proof approaches.

Data Analysis

The data obtained were analysed to reveal which proof approach the participants preferred to use and the reasons for their preferrence over the proof approaches. The preferrence of the participants were identified based on the similar concepts they used (Yıldırım and Şimşek, 2005). In order to increase the internal validity and reliability of the findings the direct quotations from the statements of the participants are given (Altunışık et. al., 2001).

FINDINGS

Proof approaches employed by the participants

Table 1 shows the number of participants based on their preferrence over the proof approaches in both geometrical statements given to them.

Table 1: Approaches towards proof preferred by students in two statements

	Statement 1	Statement 2
Synthetic approach	38	36
Vectoral approach	9	11
Analytical approach	2	2
None	1	2
Others	1	-

As seen in Table 1, for the first statement given the participants (38 students) mostly used synthetic approach to prove the assertion. This approach is followed by vectoral approach (9 students) and analytical approach (2 students). The same preferrence pattern was also found for the second statement. More specifically in the second statement the participants also preferred synthetical approach, followed by vectoral approach and analytical approach.

The category of “others” refers to the fact that one participant preferred both synthetic and vertical approach for the first statement. There were also students who used proof strategies which cannot be categorized under any proof approach categories. There were some students who preferred synthetic approach for the first statement but vectoral approach for the second statement.

The reasons for the preferrence of proof approaches by students

It was found that the participants preferred to employ synthetic approach to proof because they regarded it as easier to understand, more practical, used by teacher, habit, having less formulas and used in textbook. The distribution of the participants based on their reasons for using synthetic approach to proof is given in Table 2.

Table 2: Reasons for students to prefer synthetic approach towards proofs

	Statement 1	Statement 2
Habit	6	8
Easier to understand	14	10
Having less formulas	2	1
More practical	7	8
Used by teacher	8	7
Used in textbooks	1	1

As indicated in Table 2, the most frequently stated reason for using synthetic approach is that is much easier to understand. The following quotations exemplify the indications of the participants in this regard:
S 3: It is easy to understand…

S 10: It uses similarities and angles. Since these are the easiest topics to understand, the approach is also clear and easy to understand…
S 39: The solution is much more memorable…

There were also students who thought that this approach is much more practical. The following statements reflect this reason for preference of the synthetic approach:
S 17: Solution is short…
S 30: It is better to have much more practical solution …
S 31: This solution is much more practical…

Some students reported that they preferred synthetic approach because teachers preferred it. The followings show this reason:
S 7: In courses mostly this solution is used…
S 16: Our teacher uses this in the courses…
S 35: Our teacher prefers this way, but for me vectoral way is much more enjoyable…

Habit was also given as a reason for using the synthetic approach as exemplified by the following quotations:
S 1: It contains what I learned…
S 2: This way is much more familiar for me…
S 33: It is much more familiar way…

There were also students who reported that they preferred synthetic approach because it includes less formulas. The related student statements are given below:
S 6: The others contain more formulas …
S 14: When formulas are used I am confused…
S 28: What is proof? I do not even know what proof is, in the course teacher does not employ proof, but in textbooks mostly this approach is given…

Some students reported that their preferrence for the synthetic approach was due to its frequent use in textbooks as given below:
S 26: In textbooks always this solution is used…

In regard to vectoral approach the participants stated that they preferred this approach because it is much more reasonable, more attractive, more practical, easier to understand and has less formulas. The distribution of students based on these reasons is given in Table 3.

Table 3: Reasons for students to prefer vectoral approach

	Statement 1	Statement 2
More reasonable	1	-
More attractive	3	4
More practical	2	3
More understandable	2	4
Less formulas	1	-

The most frequently stated reason for preferring vectoral approach is its being more attractive as can be seen in Table 3. The following quotations exemplify this reason:
S 27: I like to deal with vectors…
S 40: I like to make transactions using vectors…
S 44: My interest in vectors is higher…

The participants also stated that they preferred vectoral approach due to its being much more practical. The following quotations show this reason:
S 22: Teacher generally uses synthetic approach, but vectoral approach helps me to understand the topics...

S 45: For me this approach is much easier to understand...
S 49: Vectoral approach makes it much more possible to see how things happen…

The vectoral approach was also preferred by the participants who regarded this approach as more practical. The following statements of the students indicate this reason:
S 17: The use of vector in this statement makes the proof much easier…
S 19: Proof with vectors is much easier…
S 43: When vector is used the procedure becomes more plain…

Vectoral approach was also considered by the participants to be much more reasonable, leading to be preferred by them. The related quotation is given as follows:
S 20: Teacher prefers synthetic approach, but vectoral approach is much more appropriate for my reasoning…

One student reported that she preferred vectoral approach because it contains less formulas as can be seen in the following excerpt:
S 36: This approach does not require the use of formulas…

The participants reported two major reasons for preferring the analytical approach, namely being easier to comprehend and being much more attractive. The distribution of students based on their reason for preferrence of this approach is given in Table 4.

Table 4: Reasons for students to prefer analytic approach

	Statement 1	Statement 2
Easier to comprehend	1	1
More attractive	1	1

One student reported that proof through analytic approach is much easier to comprehend. This quotation is given below:
S 8: This approach is much clearer and easier to comprehend…

The other one reported that she preferred this approach due to its being more attractive as can be seen in the following excerpt:
S 37: I like to work in the analytical setting…

In addition, one student stated that proofs made with either vectoral approach or synthetic approach are much easier to understand than those with analytical approach. His remarks are given as follows:
S 4: Analytic approach is much more difficult to understand, so the other two approaches can be employed…

DISCUSSION AND CONCLUSION

The study was carried out to identify the proof approaches employed in geometry by the students. The findings of the study suggest that students mostly used synthetic proof approach which was followed by vectoral proof approach and analytical proof approach. Therefore, less preferred proof approach was found to be vectoral approach.

One of the reasons for preferring syntehetic proof approach by the participants was its frequent use by teachers in the courses. In addition, it is thought that students much frequently come across this approach in their daily life. Due to all these reasons they thought that proofs with this approach is much easier to understand. Therefore, the use of this approach becomes a habit for them. Harel and Sowder (1998) analysed proof schemas used by students. They concluded that those students who have external proof schema use it because teachers use this schema or this schema is commonly employed in textbooks.

The reasons for preferring vectoral proof approach reported by the participants are found to be as follows: it is novice, more attractive and proofs done by it are easier to understand. Hangül and Üzel (2010) analysed the effects of computer-assisted teaching on the attitudes of the eighth graders and attempted to reveal their views about computer-assisted teaching. It was found that concepts taught in the course lasted for a long time due to the software program used in teaching these concepts. The software program was novice and included vivid and colorful techniques, leading to long-lasting concepts.

It was also found that easier proofs to understand is among the significant factors affecting the students' preferrence over the proof approaches. In addition, it was also observed that those proof approaches such as synthetic approach and vectoral approach were preferred by the participants because these approaches include less formulas and are much more practical. Of them the latter seems to influential in their preferrences due to the students' tendency to prefer practical solutions to the problems.

In some cases the participants preferred to use synthetic approach for the first statement given, but they used vectoral approach for the other one. This finding suggests that the students' preferrence over the proof approaches may vary based on the case at hand. İpek and Okumuş (2012) studied the representations used by student basic education math teachers in problem-solving. It was found that the student teachers may use differential representations for each step in solving the same problem.
Based on the present findings it can suggested that teachers should avoid using single way of teaching in classroom activities and should employ a variety of techniques, taking into account the individual differences among students.

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