International Journal on New Trends in Education and Their Implications
April 2014 Volume: 5 Issue: 2 Article: 19 ISSN 1309-6249

MATHEMATICAL KNOWLEDGE AND THE COGNITIVE AND METACOGNITIVE PROCESSES EMERGED IN MODEL-ELICITING ACTIVITIES

Dr. Juhaina Awawdeh SHAHBARI
Al-Qasemi Academic College of Education
The College of Sakhnin
Cana- ISRAEL

Assoc. Prof. Dr. Wajeeh DAHER
Al-Qasemi Academic College of Education
Nazareth- ISRAEL

Dr. Shaker Rasslan
Al-Qasemi Academic College of Education
Ibilline- ISRAEL

ABSTRACT

The study investigates the relationship between mathematical knowledge and cognitive and metacognitive processes exhibited by 83 students from Grades 6, 7, and 8 who engaged in a set of model-eliciting activities in groups of 4-5 students each. The data sources include audiotapes of their group work, worksheets, and notes. The findings indicate that the groups in each grade use different mathematical concepts. While they employed cognitive and metacognitive processes, these differed in number and distribution. The highest percent of cognitive processes and lowest percent of metacognitive processes occurred amongst the Grade 6 students. The lowest percent of cognitive processes and highest percent of metacognitive processes occurred amongst the Grade 8 students. The Grade 6 students' metacognitive processes indicate that they exhibited greater awareness than regulation and evaluation skills. Conversely, the Grade 7 and 8 students employed more regulation and evaluation processes.

Key Words: Cognitive processes, metacognitive processes, model-eliciting activities.

INTRODUCTION

Cognitive and metacognitive thinking processes—such as translation, organizing, prediction, and evaluation—are essential components of today's dynamic and technological age. Modeling activities give students an opportunity to put these processes into practice (Lesh & Zawojewski, 2007). Although broad studies have examined cognitive and metacognitive thinking processes and model-eliciting activities, little is still known about the interaction between mathematical knowledge and the nature of the cognitive and metacognitive thinking processes employed in model-eliciting activities. The present study seeks to explore this area by comparing how students from different grades and with different mathematical knowledge dealt with a specific set of model-eliciting activities.

THEORETICAL BACKGROUND

Cognitive and metacognitive processes

Cognition is a mental process or representation that manifests itself in such things as problem solving, learning memory, and reasoning (Dunlosky & Metcalfe, 2009). The first to define metacognition was Flavell—who identified as referring to "one's knowledge concerning one's own cognitive processes and products or anything related to them" (1976: 232). In a later work, he simplified this definition to "thinking about thinking" (Flavell, 1979). According to Brown (1987), metacognition includes awareness of one's own knowledge. Jacobse and Paris (1987) identify three metacognitive thinking processes—monitoring, planning, and evaluation. Monitoring includes sequential self-testing and awareness of one's comprehension and task performance. Planning involves prediction, selection of appropriate strategies, and their implementation in an optimal sequence to ensure the best allocation of resources and time. Evaluation refers to the ability to assess the efficiency of outcomes. Similar categorization suggested by Wilson and Clark (2004) they suggested three metacognitive functions- awareness, evaluation and regulation; metacognitive awareness that relates to individuals' awareness of their problem solving process, their content-specific knowledge, knowledge about their problem solving strategies. The metacognitive evaluation relates to judgments made regarding individuals thinking process, capacities and limitations as these are employed in a particular situation or as self-attributes. The metacognitive regulation relates to individuals using skills to direct their knowledge and thinking that containing regulation for the using of particular strategies and decision-making skills such as planning and setting goals.

Distinguishing between the cognitive and metacognitive processes involved in problem-solving, Garofalo and Lester (1985) argue that cognition relates to doing and metacognition to choosing and planning what to do and monitoring what is being done. Kluwe (1987) distinguishes between cognitive skills—remembering things learned earlier that might help with the present task or problem—and metacognition—monitoring and regulating the process of problem-solving. Lester, Garofalo, and Kroll (1989) maintain that cognitive processes focus on doing, reading, drawing, and calculating while metacognitive processes relate to planning, selecting, predicting, and monitoring performance. Artz and Armour-Thomas (1992) identify four cognitive processes—reading, exploring, implementing, and verifying—and six metacognitive processes—understanding, analyzing, exploring, planning, implementing, and verifying.

Many others, however, regard the distinction between cognitive and metacognitive processes as problematic (cf. Lesh & Zawojewski, 2007; Magiera & Zawojewski, 2011). Such scholars maintain that during problem-solving interactions cognitive and metacognitive processes are parallel and interactive rather than sequential. Cognition is thus inherent in metacognitive activity, while metacognition may be present in many cognitive activities.

Cognitive and metacognitive processes are also measured via different methods, Schoenfeld (1985) suggests dividing problem-solving protocols into segments of consistent behavior he calls episodes, which can then be classified according to the type of thinking they use. Many studies promote the "think-aloud" strategy (cf. Jacobse & Harskamp, 2012). Lesh, Lester, and Hjalmarson (2003) suggest a models and modeling perspective (MMP) according to which thinking becomes metacognitive when a person shifts from "thinking with" to "thinking about"—i.e., monitoring, controlling, and regulating. On this understanding, the two forms constantly interact with and influence one another.

Model-eliciting activities (MEAs): The modeling approach has become increasingly popular in mathematics education in recent decades (NCTM, 2000). A MEA is designed to a reflect real-life situation, containing incomplete, ambiguous, or undefined information regarding a problem that requires solving (English & Fox, 2005). Students must interpret and make sense of the situation in a meaningful way, the challenge encouraging them to elicit conceptual tools which function as mathematical models. This is not a linear process, the givens being tested and iteratively revised (Lesh & Harel, 2003) through multiple cycles of translation, description, data prediction, and deliverables. This is known as "mathematizing the situation" (Lesh & Doerr, 2003). MEAs call for small groups in which social interaction facilitates the development of metacognitive processes (Zawojewski & Lesh, 2003). According to Lesh, Hoover, Hole, Kelly and Post (2000), productive MEAs are based on six principles: model construction, reality, self-assessment, construct documentation, construct/shareability/reusability, and effective prototype.

International Journal on New Trends in Education and Their Implications
April 2014 Volume: 5 Issue: 2 Article: 19 ISSN 1309-6249

Research goal and questions

The present study seeks to examine the nature of the metacognitive processes used by students and compare the cognitive and metacognitive processes employed by elementary and secondary school students while working through an MEA:

1. How do elementary and secondary school students compare with respect to their use of cognitive and metacognitive processes?
2. What is the nature of the metacognitive processes employed by small groups engaged upon a specific MEA?

METHOD

The study is based on qualitative research employed to reveal which cognitive and metacognitive processes the students used while engaged upon a specific MEA.

Participants

The study was conducted in three heterogeneous classes in Grades 6, 7, and 8 in schools in an Arab village in the north of Israel. Grade 6 was represented by 26 students (11-12 year-olds), Grade 7 by 32 (12-13 year-olds), and Grade 8 by 25 students (13-14 year-olds). The students in the three classes worked in heterogeneous groups of 4-5 students each.

Procedure

Each group was given two MEAs—the "Garage" followed by the "Snow White and the Seven Dwarves". No time limits were set. The average time spent on the two MEA was approximately three lessons (each lesson = 45 minutes).

The MEAs

The two MEAs were designed on the basis of the six principles outlined by Lesh et al. (2000). The "Garage" activity*:* the owner of a toy store has some customers who wish to purchase individual parts of the set, which consists of five items. Despite all being similar in composition, the sets are priced differently. The price of the individual parts must equal the price of the whole set. As employees, the students are assigned with the decomposition and pricing task. The task sequence contains:

- Pricing the individual items when the set sells at $50.
- Pricing the individual items when the set sells $60.
- Comparing the price of the same item in steps 1 and 2.

In the "Snow White and the Seven Dwarves" set MEA, a toy store is stuck with a lot of unsold "Snow White and the Seven Dwarves" sets. This being sold in various materials—canvas, plastic, wood, carton, etc.—the prices differ. Each set consists of twenty items: a house, a Snow White, seven dwarves, and other items. Looking for a way to sell the sets, the store owner asks his workers for suggestions, adopting that of separating the sets and selling each item separately. As store employees, they students are assigned the decomposition and pricing task. The task sequences resemble those of the "Garage" MEA ($60, $80, and $90).

Data collection

The data sources include audio transcripts of the groups' discussions, two groups in each class being selected randomly and taped for the full time they worked on the problems. They also include also the groups' worksheets and notes.

Data analysis

We used open coding analysis of the conversational statements, a method that allows new categories to emerge from the groups protocols (Corbin & Strauss, 2008). The coding scheme is adapted from Kim, Park, Moore, and Varma (2013) study, which distinguishes between metacognitive activities (thinking about) and cognitive activities (thinking with)—cognitive processes not involving any evaluation or regulation and metacognitive processes being consisting primarily of evaluation and regulation. Following the categorization process, we compared the cognitive and metacognitive processes exhibited by the students in the three Grades. The categorization of the metacognitive processes was according to Wilson and Clarke (2004) method that was described in the theoretical background section.

FINDINGS

We will present the types of cognitive and metacognitive processes employed by the groups, focusing upon the essential modeling phases through their engagement in the "Garage" MEA. Subsequently we present a brief summary of their engagement in the "Snow white and the dwarves" MEA.

Grade 6 engagement in the MEAs

The two Grade 6 groups succeeded in building mathematical models for the two activities. All the groups employed fraction knowledge. As indicated by the students' discussion during the activity, the modeling process and construction of the mathematical models included both cognitive and metacognitive processes—the latter being divided into three types. We present the essential modeling phases, focusing on the cognitive and metacognitive processes.

Grade 6: The "Garage" MEA

In the first phase and after reading the first activity, the students attempted to translate it into a real context. Their first step was to name the five items. Although the items differed, the garage remained constant in all the groups' records. Doing so, the students were involved with cognitive as well as meta-cognitive processes. The cognitive processes that the students were involved with included mentioning the real life conditions of the activity (selling each item of the set separately); mentioning the real life constituents of the mathematical situation (the components of the set); and replacing a constituent of the mathematical situation with another. The meta-cognitive processes included regulating towards real life/mathematical actions regarding the separation of the set into five items; being aware of the analysis of the mathematical situation (thinking about items more relevant for the set).

In the second phase, the students priced the items in the "Garage" set when it was sold at \$50. In the beginning, they intuitively priced the items equally. After validating their solution by thinking of real examples, however, they adopted an unequal division, ranking the items according to size and importance and setting an appropriate a price for each item. The students in this phase were also involved with cognitive as well as meta-cognitive processes. The cognitive processes included: mathematical actions (computing the cost of each set's item - dividing \$50 by 5 to get \$10); giving values relevant for the mathematical situation (giving the price of the garage and the car); making conclusions regarding the mathematical situation (the price of the screwdriver and the wheel as a result of the price of the garage and car). The meta-cognitive processes included evaluating the mathematical action suggested by other students (not accepting the division of \$50 by 5); regulating for a mathematical action relevant for the mathematical situation (giving the garage a cost more than the other items of the set); being aware to the pricing process; evaluating a mathematical action (not accepting that the price of the garage is equal to the price of the screwdriver); regulating for a mathematical action (the garage should be the most expensive); awareness to the pricing process.

In the third phase, the students discussed the implications of the difference in sale price (\$50 vs. \$60). In order to compare the price of the same item in the two cases, they examined each item price in the two sets and the price relative to the other items. This demanding that they evaluate the fairness of the pricing process, they realized that the prices had to be proportionate not only within the same set but also relatively (with regard to the other sale prices). This led them to conclude that each item in the cheaper set should be cheaper than the same item in the more expensive set. In the third phase, the students performed cognitive as well as meta-cognitive processes. The cognitive processes included comparing between two mathematical situations (the two prices of every item in the two sets), while the meta-cognitive processes included awareness to the comparing process; an evaluation of a mathematical action (not accepting that the wheel's prices in the two sets are equal). Computing the relative price of each item from the overall price of the set enabled the students to work with fractions and compare the sets, leading to the construction of a fair price model. The students built a mathematical model based on fraction knowledge. The following mathematical model suggested by one of the groups exemplifies a general pricing model:

Garage = $\frac{1}{2}$ of the price, car = $\frac{1}{6}$ of the price, screwdriver = $\frac{1}{12}$ of the price, mechanic = $\frac{1}{6}$ of the price, wheel = $\frac{1}{12}$ of the price.

Computing the relative price of each item from the overall price of the set, the students performed primarily the following cognitive process: mathematical actions (building mathematical model representing the relative price of every item from the whole set). They also performed the following meta-cognitive processes: being aware to the fractional part that each item get; evaluating the mathematical situation (evaluating the suggested mathematical model); and regulating for mathematical actions (suggesting how add fractions from different denominators).

Grade 6: The “Snow White and the Seven Dwarves” MEA

While the solution process the Grade 6 groups employed was similar to the one they applied to the “Garage” MEA, here the students adduced unequal prices for the items right from the beginning. Rather than engaging in the pricing process in all the tasks, in pricing the second set they decided to construct a model that would enable them to price all the sets. This expanded the mathematical model they constructed in the “Garage” MEA. Doing so, they performed cognitive as well as meta-cognitive processes; the cognitive processes they performed were of the type of performing mathematical actions (giving specific prices for the new items, computing the overall price of all the items, giving relative prices for the new items). The meta-cognitive processes the group of students performed were regulating for mathematical actions (requesting the group to choose a large denominator and an appropriate fractional part for each item, specifying which item should have the biggest fraction: the house); being aware of mathematical actions (the price of each item, the ranking of the price items, the relative price of each item) and evaluating the mathematical actions (pricing suggestion, the ranking of items' prices).

Grade 7 engagement in the MEAs

The two Grade 7 groups succeeded in constructing two mathematical models appropriate to the two MEAs, based on percent-concept knowledge. As the students’ discussion reveals, the modeling process and construction of the mathematical models included cognitive and metacognitive processes that fell into three types. We present the essential modeling phases, focusing on the cognitive and metacognitive processes.

Grade 7: The "Garage" MEA

After reading the “Garage” MEA, the students identified the individual garage items. Doing so, they performed meta-cognitive and cognitive processes. The cognitive process was performing a mathematical action (specifying the garage constituents). While the meta-cognitive actions were: regulating for a mathematical action (imagining a real garage); being aware of the solution (the garage's items); evaluating the suggestion (the garage's items' prices).

In the second phase, the students considered the case when the garage was priced at $50, immediately pricing each individual item unequally. Doing so, they also performed cognitive and meta-cognitive processes. The cognitive processes were mathematical actions (giving a price for the set items, computing the overall price), while the meta-cognitive processes were: regulating for a mathematical action (suggestion of the ranking of the prices); being aware of the pricing process (the price of each item comparing to other items and to the overall price); evaluating a mathematical action (accepting the pricing action, because it fulfilled the condition "the most important is the most expensive").

Afterwards, the students considered the situation when the set was priced at $60, and used the model they had arrived at for the $50 set. Doing so, the students mainly performed meta-cognitive and cognitive processes. The cognitive processes were a mathematical actions (computing the items' price, comparing the items' prices). The meta-cognitive processes were: regulating for a mathematical action (suggesting to add $2 for each item, to divide the price according to the items' importance or to compare with a previous mathematical model) and evaluating (not accepting the adding strategy, accepting the relative division).

In comparing the price of the items in the two sets, the students understood that they should have the same ratio. Doing so, the cognitive processes that they performed were: performing mathematical action (pricing the two sets; concluding regarding the relative price of the items, for example the garage is 40% of the set). In addition, the meta-cognitive processes that they performed were: being aware of the pricing process (comparing the items' prices, the relative price of each item); regulating for considering a mathematical relation (suggesting that the items' prices should be the same percent in both sets); evaluating mathematical actions (comparing of items' prices, the suggestion of the comparing process).

The comparison between the items' prices in the two sets prompted the students to use their percent knowledge to construct a mathematical model that functioned as a pricing model: garage = 40% of the set, car = 30% of the set, worker = 10% of the set, crane = 10% of the set, wheel = 10% of the set. Doing so, the students used meta-cognitive and cognitive processes. The meta-cognitive processes were regulating (suggesting the use of percentage in giving prices to items); evaluating the suggestion. while the cognitive processes were mathematical actions that resulted from following the regulating, i.e. giving prices to the items as percentages of the set.

Grade 7: The "Snow White and the Seven Dwarves" MEA

The Grade 7 students first identified the individual items of the "Snow White" set. In place of the pricing process they had originally employed in the "Garage" MEA they adopted the mathematical model they had constructed during this activity. Doing so, they performed meta-cognitive processes and cognitive processes; The cognitive processes were mathematical actions (pricing the items using percent; computing the price of the all items). The meta-cognitive processes were: regulating for a mathematical action (Doing the same as in the previous set), being aware of the pricing process and evaluating the mathematics situation (The Snow White is the most important item).

Grade 8 engagement in the MEAs

The two Grade 8 groups succeeded in constructing two mathematical models to fit the two activities. They used decimals, ratio and proportion, and percents, building their general models on the basis of the percent concept. As demonstrated in their discussions, the modeling processes and construction of the mathematical models included cognitive and metacognitive processes, divided into three principal types. We shall present the essential modeling phases, focusing on the cognitive and metacognitive processes.

Grade 8: The "Garage" MEA

After reading the "Garage" MEA, they identified the individual items, performing meta-cognitive processes and cognitive processes. The cognitive processes were mathematical actions (specifying the garage constituents). The meta-cognitive processes include regulating for a mathematical action (imagining a real garage) and evaluating the solution.

In the second modeling phase the students were engaged in pricing the first set, they adopted an unequal division based on the items' importance. Doing so, the students were involved with cognitive as well as meta-cognitive processes. The cognitive processes were giving a price for the set's items. The meta-cognitive processes were: regulating for a mathematical action relevant for the mathematical situation (giving the garage a cost more than the other items of the set). In the pricing of another set, the meta-cognitive processes were: regulating for a mathematical action (suggesting computing the ratio of each item from the overall price in the first set and then to generalize to the second set), and evaluation of the suggestion. The cognitive processes included the calculating the ratio of each item in relation to the overall price.

In comparing the prices of the two sets, the students used relative ratios, expressing their opinion using the percent representation. They stressed the fact that their mathematical model was appropriate for all the sets: Doing so, the cognitive processes that they performed were: performing mathematical actions (pricing the two

International Journal on New Trends in Education and Their Implications
April 2014 Volume: 5 Issue: 2 Article: 19 ISSN 1309-6249

sets; concluding regarding the relative price of the items, for example the car is 20% of the overall price). In addition, the meta-cognitive processes that they performed were: regulating for a mathematical relation (suggesting that the items' prices should be the same percent in both sets) and evaluating their solution (the relevant of using the ratio in the pricing process).

Grade 8: The "Snow White and the Seven Dwarves" MEA

The mathematical model suggested in the first activity was expanded and implemented in the "Snow White and the Dwarves" MEA. Rather than seeking to arrive at a pricing model for the sets they directly suggested a mathematical model. Doing so, they performed meta-cognitive processes and cognitive processes. The cognitive processes they performed were of the type of performing mathematical actions (ranking the sets' items, giving relative prices for the set's items; computing the sum of all the prices items). The meta-cognitive processes performed were: regulating for mathematical actions (suggesting the use of the pricing model from the previous activity, suggesting the use of percentage, and suggesting the ranking of the price), being aware to the pricing process (the relative price of each item, the overall price of several items) and evaluating mathematical actions (pricing suggestion, ranking of the items' price).

Cognitive and metacognitive processes across the three grades

The finer grained analysis of students' conversational statements through their engagement in the two MEAs indicate that cognitive and metacognitive processes that students evinced were integrated as clarified in the following segment of students' discussion through their engagement in the "Garage" activity:

43. Mahmood: $50 divided by 5, so each item will be $10 [*cognitive*]
44. Samer: (recording) 50:5=10 [cognitive]
45. Haia: Why are you dividing them this way? [*metacognitive* (*evaluating*)]
46. Nadeen: The garage must be the more expensive item. [*metacognitive* (*reguation*)]
47. Haia: we must try to divide the items unequally [*metacognitive* (*regulating*)]
48. Samer: The garage is $15, the car is $10 … [*cognitive*]
49. Nadeen: No, the garage cannot be the same price as the screwdriver [*metacognitive* (*evaluating*)]

metacognitive processes the students evinced in the two modeling activities were similar in type across the three grades. Analysis of their discussions reveals the presence of other processes unrelated to the situation solution—such as "We have two cars" or "I go with my father to the garage." These segments were coded as irrelevant. Through The percentage distribution of the cognitive and metacognitive processes is presented in Table 1.

Table 1: Percentage distribution of cognitive and metacognitive processes in the two MEAs

Process / Grade		Metacognitive
6	51%	37%
7	46%	39%
8	43%	45%

The metacognitive processes were divided into three types—regulating, awareness, and evaluating. The distribution of these three types is presented in Table 2.

Table 2: Percentage of the metacognitive processes types per grade in the two MEAs

Process / Grade	Metacognitive		
	Regulation	Awareness	Evaluation
6	25%	45%	28%
7	36%	33%	30%
8	33%	31%	35%

International Journal on New Trends in Education and Their Implications
April 2014 Volume: 5 Issue: 2 Article: 19 ISSN 1309-6249

DISCUSSION

The study examines the relationship between mathematical knowledge and cognitive and metacognitive processes as evidenced by primary and middle-school students engaged in two MEAs. To do so, primary and middle schools groups of student carried out two MEAs that had the same mathematical structure, where all the Grade groups constructed a mathematical model for each activity, the models being based on various mathematical concepts and knowledge. The overall findings confirm numerous other studies that have suggested that MEAs trigger and foster metacognitive processes (Lesh & Zawojewski, 2007; Magiera & Zawojewski, 2011). Small-group frameworks induce social interactions that encourage regulatory processes and metacognitive monitoring (Goos, Galbraith, & Renshaw, 2002), also giving students the opportunity to share their mathematical models and test and revise their assumptions and predictions (Lesh & Doerr, 2003).

Comparing the different grades models, the grade 6 students used fractions, the grade 7 students used decimals and percentages, and the grade 8 students used proportional ratios and percentages. This finding contrasts with Mousoulides, Sriraman, Pittalis, and Christou (2007) comparison of Grade 6 and 8 students, which reported that Grade 6 students failed to effectively apply mathematical processes in MEAs while grade 8 students succeeded in doing so. In our research, what made the three grades succeed in getting a mathematical model is that the activity was appropriate for the three grades.

Analysis of the students' cognitive and meta-cognitive processes while carrying the MEAs indicates the integration of the two types of processes in students' work — a result that confirms the findings of previous studies (Lesh and Zawojewski, 2007; Magiera & Zawojewski, 2011) pointing that cognition and metacognition are essentially integrated during problem-solving tasks, where they develop interactively and in parallel. Our findings also support Artz and Armour-Thomas's (1992) claim that a continuous interplay of cognitive and metacognitive behaviors appears to be necessary for successful problem solving and maximum student involvement.

The largest percent of cognitive processes were evinced by the grade 6 groups. Artz and Armour-Thomas (1992) suggest that groups that employ more cognitive than metacognitive process are frequently less successful than other groups. In the present study, the greater use of cognitive processes by grade 6 groups may be due to a lack of ability to generalize and plan. In contrast to the groups in the other two grades, who directly adopted an unequal pricing system, grade 6 students initially priced the items equally. Similarly, coming to the second MEA "Snow White and the Seven Dwarves", grade 6 students initially priced the items in the MEA, i.e. they repeated the steps of the first MEA again, and only subsequently they reached a general pricing model. Grade 7 groups extrapolated a model from their pricing of the first set, while the Grade 8 students adopted a general pricing model immediately.

With respect to metacognitive processes (awareness, regulation, and evaluation), the findings indicate that grade 6 students used the smallest, while grade 8 the greatest percent**.** This finding supports Bryce and Whitebread's (2012) study, according to which, while metacognitive processes improve with age and task-specific ability, metacognitive deficiencies are more affected by task-specific ability than by age. Analysis of the Grade 6 students' metacognitive processes indicates that they showed greater awareness than regulation and evaluation skills; in contrast to Wilson and Clarke (2004) study who reported that awareness was the less reported metacognitive process. The results of the previous study agree with our findings regarding grade 7 and 8 students' use of metacognitive processes. These students employed more regulation and evaluation process, where evaluation was the most commonly used amongst the three metacognitive processes, a finding that also supports Wilson and Clarke's (2004) study. The principal difference in metacognitive processes distribution was observed between the grade 6 and middle school groups, where grade 7 and 8 students' metacognitive processes evinced a much closer distribution. The latter finding supports Leutwyler's (2009) report that little development in metacognitive skills occurs between grades 10 and 12, where in our case little difference was observed between grade 7 and grade 8.

International Journal on New Trends in Education and Their Implications
April 2014 Volume: 5 Issue: 2 Article: 19 ISSN 1309-6249

IJONTE's Note: This article was presented at 5th International Conference on New Trends in Education and Their Implications - ICONTE, 24-26 April, 2014, Antalya-Turkey and was selected for publication for Volume 5 Number 2 of IJONTE 2014 by IJONTE Scientific Committee.

BIODATA AND CONTACT ADDRESSES OF AUTHORS

Juhaina Awawdeh SHAHBARI is a Doctor of Mathematics Education at the University of Haifa, Al-Qasemi Academic College of Education and the College of Sakhnin. She received her master's degree and doctorate at the University of Haifa in Israel in the mathematics education department.

Her research interest is in mathematics education: modeling, realistic mathematics education, instruction design, common misconceptions in mathematical concepts.

Dr. Juhaina Awawdeh SHAHBARI
Cana- ISRAEL
16930 P.O.B 712
E. Mail: juhaina8@gmail.com

Wajeeh M. DAHER is an associate professor at An-Najah National University and senior lecturer at Al-Qasemi Academic College of Education. He serves as the department chair of the M.Ed. program in mathematics education at Al-Qasemi Academic College of Education.

His main research interest is mathematics education, in addition to alternative methods in teaching and learning, technology in education and the different aspects of learning: cognitive, meta-cognitive, affective, meta-affective and social.

Assoc. Prof. Dr. Wajeeh M. DAHER
st. Motran 5092 92
Nazareth- ISRAEL
E. Mail: wajeehdaher@gmail.com

Shaker A. RASSLAN is a Doctor of Mathematics Education at AL-Qasemi- Academic College of Education. He served as a lecturer, a researcher, and a supervisor in Mathematics Education for the M.Ed. program.Dr. Rasslan has M.Sc Degree in Mathematics Education at The Department of Education for technology and Sciences; Technion; Israel; Ph.D (1996) Degree in Mathematics Education at Hebrew University of Jerusalem; ISRAEL and Post Doctoral (1999) researcher in Mathematics Education at The Mathematics Education Research Center at Warwick University, UK. His research interest is Cognitive Development, Concept Image, Advanced Mathematical Thinking, Creativity in Mathematics and Effective Mathematics under Computers Environment. He has authored, co-authored or presented over twenty years many chapters, articles and conference presentations, As well; he is a reviewer for research papers in the International Group for the Psychology of Mathematics Education (PME) as well as in PME38 in Vancouver- Canada.

Dr. Shaker A. Rasslan
30012, Ibilline, ISRAEL
E.Mail: shaker1953@gmail.com

REFERENCES

Artz, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction, 9*(2), 137-175.

Brown, A. L. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 65-116). Hillsdale, NJ: Erlbaum.

Bryce, D., & Whitebread, D. (2012). The development of metacognitive skills: Evidence from observational analysis of young children's behavior during problem-solving. *Metacognition Learning, 7*, 197-217.

Corbin, J., & Strauss, A. (2008). Basics of qualitative research (3rd ed). Thousand Oaks, CA: Sage.

Dunlosky, J., & Metcalfe, J. (2009). Metacognition. Los Angeles: Sage.

English, L. D., & Fox, J. L. (2005). Seventh-graders' mathematical modelling on completion of a three-year program. In P. Clarkson et al. (Eds.), *Building connections: Theory, research and practice* (Vol. 1, pp. 321-328). Melbourne: Deakin University Press.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231–236). Hillsdale, NJ: Erlbaum.

Flavell, J. H. (1979). Metacognition and cognitive monitoring; A new area of cognitive developmental inquiry. *American Psychologist, 34*(10), 906-911.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49*(2), 193-223.

Jacobse, J. E., & Harskamp, E. G. (2012). Towards efficient measurement of metacognitive in mathematical problem solving. *Metacognition Learning, 7*, 133-149.

Jacobse, J. E., & Paris, S. G. (1987). Children's metacognition about reading: Issues in definition, measurement, and instruction. *Educational Psychologist, 22*, 255-27.

Kim, Y. R., Park, M. S., Moore, T. J., & Varma, S. (2013). Multiple levels of metacognition and their elicitation through complex problem-solving tasks. *Journal of Mathematical Behavior, 32*(3), 377-396.

Kluwe, R. H. (1987). Executive decisions and regulation of problem solving behavior. In F. Weinert, & R. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. (31-64). Hillsdale, New Jersey: Lawrence Erlbaum.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum.

Lesh, G., & Harel, R. (2003). Problem solving, modelling and local conceptual development. *Mathematical Thinking and Learning, 3*(2/3), 157-190.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *The handbook of research on mathematics teaching and learning* (2nd ed.). (pp. 763 - 804). Reston, VA/Charlotte, NC: National Council of Teachers of Mathematics.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought revealing activities for students and teachers. In R. Lesh & A. Kelly (Eds), *Handbook of research design in mathematics and science education* (pp. 591- 644). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., Lester, F. K., & Hjalmarson, M. (2003). A models and modeling perspective on metacognitive functioning in everyday situations where problem solvers develop mathematical constructs. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 383-403). Mahwah, NJ: Lawrence Erlbaum.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). *The role of metacognition in mathematical problem solving: Final Report*. Bloomington, IN: Indiana University (Eric Document Reproduction Service No. ED 3 t 4 255).

Leutwyler, B. (2009). Metacognitive learning strategies: Differential development patterns in high school. *Metacognition Learning, 4*, 11-123.

Magiera, M. T., & Zawojewski, J. (2011). Characterizations of social-based and self-based contexts associated with students' awareness, evaluation, and regulation of their thinking during small-group mathematical modeling. *Journal for Research in Mathematics Education, 42*(5), 486-520.

Mousoulides, N., Sriraman, B., Pittalis, M., & Christou, C. (2007). Tracing students' modeling processes in elementary and secondary school. *Proceedings of the Fifth Conference of the European Society for Research in Mathematics Education* (Thematic Group 13). Cyprus: Larnaca.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Schoenfeld, A. H. (1985). Mathematical problem solving. Orlando, FL: Academic.

Wilson, J., & Clarke, D. (2004). Towards the modelling of mathematical metacognition. *Mathematics Education Research Journal, 16*(2), 25-48.

Zawojewski, J., & Lesh, R. (2003). A models and modeling perspective on productive problem solving strategies. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modeling perspective on problem solving, learning and instruction in mathematics and science education* (pp. 317-336). Mahwah, NJ: Lawrence Erlbaum.