

PRE-SERVICE PRIMARY MATHEMATIC TEACHERS' SKILLS OF USING THE LANGUAGE OF MATHEMATICS IN THE CONTEXT OF QUADRILATERALS

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ABSTRACT

Mathematics is defined as a discipline which has a specific language that encompasses a variety of symbols, figures, and terms and which depends on the relationship among these properties. Effective learning and teaching of mathematics considerably depends on the accurate use of language. In line with these, this study aims to explore pre-service primary mathematics teachers' skills of defining and expressing a variety of concepts and properties in the context of quadrilaterals using the language of mathematics. Another objective is to determine pre-service mathematics teachers' opinions on using language in mathematics teaching and explore the relationship between their opinions and their skills of using the language of mathematics. For this aim, data collection tools include the "Language in Mathematics Teaching Scale" and a test which involves six open-ended questions that cover the sub-dimensions of defining a concept, expressing conceptual properties with a verbal and a symbolic language, and using symbols to represent the properties of shapes. A variety of categories were created in line with the fourth grade pre-service primary teachers' responses to the open-ended questions. Percentage and frequency values were therefore determined and the data obtained from the scale were interpreted with the help of the SPSS programme. The findings were discussed in line with the literature and some suggestions were presented.

Keywords: Language of mathematics, quadrilaterals, pre-service teacher.

INTRODUCTION

Even though it is hard to define mathematics, which is a broad concept, it can be basically defined as a language which is composed of systems and patterns (Goldenberg, Cuoco & Mark, 1998) that use certain symbols, signs, numbers and shapes to prove the relationship among these representations (Reysi Suydam, Lindquist, & Smith 1995). The research studies in the literature involve different approaches to mathematics. Some suggest that math is a universal language. Others suggest that it has its own language, and the rest suggest that it is difficult to define mathematics (Moschkovich, 2012). Using the language of mathematics means a lot more than possessing its vocabulary. It is necessary but not sufficient to know the mathematical terms and concepts because the language of mathematics encompasses defining patterns, making generalizations and supporting what is meant with representations (Moschkovich, 2012). Meaney (2005) groups the language of mathematics under three headings: everyday language, mathematical concepts and symbolic language. Learning takes place when students can make a connection between these three fields and make transitions. This structure of the language of mathematics enables students to represent their understanding of mathematics and convey the results they come up with (Ní Ríordáin, 2009).

Although mathematics is a widely-used instrument in the daily life, school is the place where one encounters the use of language in mathematics in the broadest sense (Başaran,1998). Students have to learn the language of mathematics in order to work on the problems, organize their thoughts relying on a particular structure and express them in a clear and comprehensive way, understand their own and others' mathematical thinking and problem solving processes, and gain flexibility in their skills to interpret and express their thoughts (National Council of Teachers of Mathematics, 2000). Many researchers argue that it is essential for students to use the language of mathematics because those who can use it effectively, in other words, who can read, write and comprehend the mathematics contents, can understand mathematics better and achieve in this discipline (Buchanan, 2007). The best way for students to learn mathematics is to use it. Understanding the language of mathematics enables students to develop their skills of internalizing newly introduced concepts, thinking over and discussing them (Chard, 2003). In addition to this, the language of mathematics is also important for students to attach meanings to the newly introduced mathematical concepts and interpret the differences among them.

Success in mathematics teaching is directly linked to the accurate use of the language of mathematics (Ferrari-Luigi, 2004; Pimm, 1987). Therefore, it is necessary to pay attention to several properties (level suitability, accurateness, ability to represent concepts, usage etc.) of the language in a classroom environment (Bali-Çalikoğlu, 2002). The way of understanding mathematics is closely linked to expressing mathematical concepts accurately and using mathematical terms properly (Buchanan, 2007). A teacher should pay attention to their language of mathematics so as not to cause misinterpretations in students and also take into account the language of students. Therefore, the students have the opportunity to fix their errors (if any) while using the language of mathematics and develop their skills. At this point, while Glaserfeld (1995) argues that the interaction between the teacher and the student is important, Gökkurt, Soylu, Gökkurt (2012) argue that teachers need to transfer the way they understand mathematical concepts, express their definitions, explain their analyses, and ensure that the students have an opportunity to comment on the mathematical topics they have learned. Because when students describe what they think and do, it will enable both the student himself and the teacher to assess the students in terms of mathematical understanding (Doğan & Güner, 2012). As mathematics progresses cumulatively, the language of mathematics used in this process prospers. For this reason, inaccurate and faulty usages during the formation of the basis of the language of mathematics starting from the primary level education would cause the students to carry this incorrect language into future. In this context, the language of mathematics a teacher uses at the primary to secondary level interfaces is quite important (Aydın & Yeşilyurt, 2007). A teacher has great influence on enabling students to use the accurate language of mathematics (Raiker, 2002). Teachers' knowledge is mostly formed and shaped during their education period in faculties of education (Bozkurt & Koç, 2012). Therefore, it is important to study pre-service teachers' skills of using the language of mathematics accurately and to detect their errors.

Çakmak, Bekdemir and Baş (2014) emphasize the importance of using verbal and symbolic languages, which are a part of the language of mathematics, in the classroom and showing the connections between them clearly. This would make it easy for students to understand the transition between these languages. The studies conducted prove that students have difficulty expressing the concepts with a verbal and symbolic language (Capraro & Joffrion, 2006), and fail to use the mathematical area language (Doğan & Güner, 2012; Yeşildere, 2007). In addition to this, many teachers use the language of mathematics carelessly, represent a mathematical concept with a different concept that is not corresponding such as expressing the term of amount with the term of number and confuse mathematical terms (Haylock & Thangata, 2007). The fact that students and teachers have good skills of using the language of mathematics would make it easy for teachers to see better student profiles (Larson, 2007) and increase both the teachers' and students' success.

The studies on the language use in mathematics prove that students generally face challenges in transferring mathematical situations and thoughts into the language of mathematics (Dur, 2010; Korhonen, Linnanmäki & Aunio, 2011; Rudd, Lambert, Satterwhite & Zaier 2008; Woods, 2009). Gür (2006) suggests that pre-service teachers make incorrect definitions. Orton and Frobisher (1996) suggest mathematical meaning attributed to a concept depends on a person and there is a risk of expressing a mathematical concept incorrectly based on the attributed meaning. Otterburn and Nicholson (1976) argue that even though students are familiar with

mathematical terms, they have difficulty in expressing and internalizing them. In addition to this, several research studies were conducted in this field to explore the significance of the language of mathematics (Barwell, Leung, Morgan, & Street, 2005; Forman & van Oers, 1998; Hoyles & Forman, 1995; Monaghan, 1999; Sfard, 2000; Sfard & Kieran, 2001). However, it is observed that there is little research available in Turkey when the number of studies in the other countries is considered (Aydın & Yeşilyurt, 2007; Çakmak vd, 2014; Çalikoğlu-Bali, 2003; Doğan & Güner, 2012; Göktürk, Soylu & Örnek, 2013).

Using the language of mathematics in geometry is extremely important (Toptaş, 2015) since geometry, which is a part of mathematics, is a basic skill and is necessary for communication both in the daily life and classroom environment (Sherard, 1981). At this point, a teacher's job is to pay attention to the accurateness of the language of mathematics and appropriateness to the student's level. Explaining the generalization "Square is also a rectangle" to the students before a certain level leads to a confusion in students. Giving this information to students after they construct the necessary background would make the concept more meaningful to them (Baykul, 2000; van Hiele, 1986). Geometry has three dimensions: shapes representing the concepts, definitions, and the properties. In other words, a visual image that represents a geometric concept has a definition that helps make sense of the various properties and concepts and differentiate between them (Türnüklü, Alaylı & Akkaş, 2013). The studies conducted focus on the way pre-service teachers define some geometric concepts and the language of mathematics. The results suggest that pre-service teachers fail to use the language of mathematics and define geometric concepts (Çetin & Dane, 2004; Dane, 2008; Kuzniak & Rauscher, 2007; Pickreign, 2007; Sandt & Nieuwoudt, 2003). Researchers suggest conducting similar studies on different geometric concepts and collecting detailed information for the aim of identifying the pre-service teachers' strengths and weaknesses and take precautions against them. The way of increasing students' success in geometry is parallel to the quality of mathematics and geometry education at primary level education (Pusey, 2003). The data to be collected from the steps of drawing and defining a geometric shape, explaining its properties, and exemplifying them would be guiding for understanding a teacher's proficiency (Çakmak, Konyalıoğlu & Işık, 2014). In addition to this, most of the research studies have been conducted with pre-school students, first level primary school students and pre-service teachers and not enough studies have been conducted with pre-service second level primary school teachers (Türnüklü et al, 2013). Accordingly, this study aims to identify the way primary pre-service teachers define the types of quadrilaterals, analyze the shapes they draw, examine the language of mathematics they use to express geometric concepts and properties, and to present the relationship between the pre-service teachers' views on the language of mathematics and their skills of using the language of mathematics. Thereby, the research questions that guide this study are as follows:

1. What are the pre-service primary mathematics teachers' skills of using the language of mathematics in the context of quadrilaterals?
2. What are the pre-service primary mathematics teachers' views on using language in mathematics teaching?

METHOD

Research Model

The research was designed in survey (descriptive-survey) model since it aims to investigate the current situation regarding pre-service primary mathematics teachers' skills of using the language of mathematics in the context of quadrilaterals.

Study Group

The research was conducted with 50 pre-service teachers in the 4th grade studying primary mathematics education at a state university located in a city in the north-west of Turkey during the academic year 2015-2016. This study group was formed on a volunteer basis due to the fact that senior students have taken most of the mathematics education courses and have the necessary background.

Data Collection

The data were collected using a test which consists of 6 open-ended questions related to defining quadrilateral types, expressing conceptual properties with a verbal and symbolic language, and representing these attributes by drawing shapes in an effort to explore the pre-service teachers' skills of using the language of mathematics. The participants were asked to make a definition regarding the concepts of square, rectangle, parallelogram, rhombus, trapezium, and kite (deltoid), write their features verbally, express them symbolically and represent these properties by drawing their shapes. The test questions were prepared with the help of the studies on geometric concepts in the literature and kept in a similar nature.

The "Language in Mathematics Teaching Scale" developed by Bali-Çalikoğlu (2002) was used to identify pre-service teachers' views of using the language of mathematics. This scale is designed in a five-point Likert type and the answer possibilities range between "I do not agree at all" and "I strongly agree". The reliability coefficient of the scale is 0.82. It is composed of four factors which are "written expression and written assignments", "symbolic expression", "problem formation", and "verbal expression".

Data Analysis

An SPSS statistical package program was used for the analysis of the data obtained from the "Language in Mathematics Teaching Scale", which was used to identify pre-service teachers' views of using the language of mathematics. The findings were interpreted and the significance level was taken as $p=0.05$. The responses given were labelled as 5- I strongly agree, 4- I agree, 3- I am not sure, 2- I do not agree, 1- I do not agree at all. The negative statements were inverted and included in the analysis. Arithmetic mean, standard deviation value, frequency and percentage values were calculated. The Cronbach's alpha coefficient was calculated as 0.89. The arithmetic means were interpreted as follows: 18-32.4 corresponds to "I do not agree at all", 32.4-46.8 corresponds to "I do not agree", 46.8-61.2 corresponds to "I am not sure", 61.2-75.6 corresponds to "I agree" and 75.6-90 corresponds to "I strongly agree".

The responses given to 6 open-ended questions prepared in relation to investigating pre-service teachers' skills of using the language of mathematics in the context of quadrilaterals were analyzed through content analysis method. Content analysis is a method which makes it possible to analyze, understand, organize, define and interpret the verbal and written data systematically and objectively (Sommer & Sommer, 1991). Accordingly, each teacher's responses were studied detailedly, various categories were formed, concepts included in the categories were defined and interpreted (Patton, 2002). Some research studies in the literature (Fujita, 2012; Türnüklü, Gündoğdu-Alaylı & Akkaş, 2013; Erşen & Karakuş, 2013) were made use of to form categories in order to make the data analysis more easily and the evaluation criteria were identified and used. The data were labeled separately by two researchers and the percentage of conformity was found to be %89. Researchers reached a consensus after a meeting and matched the responses and categories. Percentage and frequency values corresponding to each category and response were presented in a table. In this way the data were analyzed and the categories were made reliable.

Categories in relation to describing the quadrilateral properties using verbal and symbolic language together were formed in order to interpret pre-service teachers' skills of using the language of mathematics quantitatively. Six categories were merged into four categories. Therefore, the responses given to open-ended questions by the pre-service teachers were rated as follows: each statement in the category of "Correct Verbal Expression-Correct Symbolic Representation" was rated 3 points, the categories of "Correct Verbal Expression-Missing Symbolic Representation" and "Missing Verbal Expression-Correct Symbolic Representation" were merged and each statement in these categories was rated 2 points, "Correct Verbal Expression-Incorrect Symbolic Representation" and "Incorrect Verbal Expression-Correct Symbolic Representation" were merged into one and each statement was rated 1 point, the statements in the category of "Incorrect Verbal Expression-Incorrect Symbolic Representation" was labeled 0. It was agreed that those who got the score of 42-54 in the responses ranked as 0-3 used the language of mathematics both verbally and symbolically. Those who got the score of 30-42 used either of the verbal or symbolic languages. Those who got the score of 18-30 used either of

the verbal or symbolic language well and incorrectly used the other one. Those who got the score of 6-18 used the language of mathematics both verbally and symbolically incorrectly.

Table 1: Criteria Related to Interpreting the Language of Mathematics Used in the Context of Quadrilaterals

Criteria	Square	Rectangle	Parallelogram	Rhombus	Trapezium	Deltoid
Criteria of Describing Quadrilaterals	All sides have equal length and all interior angles are right angles (90°).	Opposite sides are of equal length and all interior angles are right angles (90°).	Opposite sides are parallel and of equal length	Opposite sides are parallel and all sides have equal length	The upper and lower sides are parallel	Formed by two isosceles triangles with congruent bases
Criteria of Drawing Quadrilaterals and Displaying Properties by Symbols	At least three corner angles are right angles (90°). Drawing a four-sided closed shape with all sides of equal length and using notations	At least three corner angles are right angles (90°). Drawing a four-sided closed shape with equal opposite sides and using notations	Drawing a four-sided closed shape with opposite sides parallel and of equal length and using notations	Drawing a four-sided closed shape with opposite sides parallel and all sides of equal length and using notations	Drawing a four-sided closed shape with upper and lower bases parallel and using notations	Drawing a four-sided closed shape formed by two isosceles triangles with congruent bases and using notations
Criteria of Explaining the Properties of Quadrilaterals Verbally & Criteria of Expressing the Properties of Quadrilaterals Symbolically Through the Figures Drawn	Specifying properties such as "sides are of equal length", "all angles are right angles" (90°), "opposite sides are parallel", "diagonals are of equal length" and "diagonals intersect vertically", and	Specifying properties such as "opposite sides are of equal length", "all angles are right angles" (90°), "opposite sides are parallel", "diagonals are of equal length", and "diagonals bisect each other",	Specifying properties such as "opposite sides are parallel and of equal length", "opposite angles are equal", "diagonals bisect each other", and "the angles on either end of each side are supplement angles" with appropriate mathematical terms verbally and writing	Specifying properties such as "opposite sides are parallel", "all sides are of equal length", "diagonals intersect vertically", "the angles in the corner where diagonals unite are bisectors" with appropriate mathematical terms verbally and writing	Specifying properties such as "the upper and lower sides are parallel", "the angles on the ends of each side which is not parallel are supplement ary angles", and "intermediate base is parallel with other bases and half the length of two bases"	Specifying properties such as "the edges converging at two opposite corners are equal by twos", "one of the diagonals is the bisector of opposite angles", "opposite angles are equal", "diagonals intersect vertically and one

	“diagonals are bisectors of interior angles” with proper mathematical terms verbally and writing them in a symbolic language	with proper mathematical terms verbally and writing them in a symbolic language	them in a symbolic language	them in a symbolic language	with appropriate mathematical terms verbally and writing them in a symbolic language	diagonal bisects the other one into two equal parts” with appropriate mathematical terms verbally and writing them in a symbolic language
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FINDINGS

Findings and Comments Regarding the Quadrilateral Test That Consists of Open-Ended Questions

Table 2. Percentage and Frequency Values Regarding Quadrilateral Definitions

Definition	Square		Rectangle		Parallelogram		Rhombus		Trapezium		Deltoid		Total	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
Correct	22	44	17	34	20	40	21	42	32	64	36	72	148	49
Faulty	15	30	14	28	19	38	13	26	1	2	5	10	67	23
Naming														
Too Many Properties	8	16	14	28	1	2	-	-	8	16	-	-	31	10
Incorrect	5	10	5	10	10	20	16	32	9	18	7	14	52	17
Blank	-	-	-	-	-	-	-	-	-	-	2	4	2	1
Total	50		50		50		50		50		50		300	

Looking at the definitions made by the pre-service primary teachers about quadrilaterals in Table 2, it is observed that 49% of the pre-service primary teachers made correct definitions, 23% of them made faulty definitions, and 17% made incorrect definitions. In addition to this, while 10 % tried to explain the concepts by mentioning the properties rather than making definitions, 1% could not define “kite” (deltoid). The findings obtained show that only half of the pre-service primary teachers expressed the types of quadrilaterals using the accurate language of mathematics. The other half, on the other hand, could not do that. It can be noted that pre-service primary teachers have inadequacies or fallacies with their knowledge about quadrilaterals. While the pre-service teachers could make the most number of correct definitions for kites (deltoid) and the least number of correct definitions for rectangles, they made the most number of incorrect definitions for rhombuses and the least number of incorrect definitions for squares and rectangles.

Table 3: Percentage and Frequency Values Regarding Quadrilateral Drawings

Drawings	Square		Rectangle		Parallelogram		Rhombus		Trapezium		Deltoid		Total	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
Correct	38	76	37	74	10	20	11	22	32	64	39	78	167	56
Faulty	11	22	10	20	36	72	35	70	13	26	3	6	108	36
Incorrect	1	2	3	6	4	8	4	8	5	10	8	16	25	8
Total	50		50		50		50		50		50		300	

Table 3 shows the classified values of the drawings made by the pre-service teachers related to the types of quadrilaterals. 56% of the quadrilateral types were correct, 36% were faulty and 8% were incorrect. Most of the pre-service teachers were observed to have faults in their drawings, and fail to display quadrilaterals using appropriate mathematical notions. This can be attributed to the fact that the pre-service teachers have difficulty reflecting the language of mathematics over the shapes and establishing the link between shape and notation. The rate of drawing correct shapes was high for the concepts of kite (deltoid), square, and rectangle, whereas it was low for the concepts of parallelogram and rhombus. The rate of drawing an incorrect shape was lowest for a square and highest for a kite (deltoid). The rate of drawing a faulty shape was the highest for a parallelogram and rhombus.

Table 4. Percentage and Frequency Values Related to Expressing the Properties of Quadrilaterals Verbally

Verbal Expression	Square		Rectangle		Parallelogram		Rhombus		Trapezium		Deltoid		Total	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
Correct	142	94	105	93	94	85	91	90	45	74	36	70	513	87
Faulty	2	1	2	2	-	-	-	-	-	-	-	-	4	1
Incorrect	8	5	5	5	16	15	10	10	16	26	15	30	70	12
Total	152		112		110		101		61		51		587	

Looking at the verbal language pre-service teachers used to explain the properties of quadrilateral types, it can be noted that the pre-service teachers use an accurate language in 87% of the quadrilateral properties, incorrect language in 12% and faulty language in 1%. The pre-service teachers were observed to express the properties of quadrilaterals with substantially correct mathematical concepts. According to the findings, the pre-service teachers best expressed the properties of a square using the correct verbal language. The worst expressed, on the other hand, were the properties of a kite. This situation might be attributed to the fact that pre-service teachers are more familiar with the properties of a square rather than a kite.

Table 5: Percentage and Frequency Values Related to Representing the Properties of Quadrilaterals Symbolically

Symbolic Representation	Square		Rectangle		Parallelogram		Rhombus		Trapezium		Deltoid		Total	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
Correct	101	77	86	80	77	68	74	80	45	66	57	84	440	76
Faulty	10	8	6	5	9	8	1	1	-	-	1	1	27	4
Incorrect	20	15	16	15	28	24	18	19	23	34	10	15	115	20
Total	131		108		114		93		68		68		582	

Table 5 shows the findings in relation to the symbolic language the pre-service teachers used to explain the properties of quadrilateral types. According to the results, the pre-service teachers used a correct symbolic language in 76% of the quadrilateral properties, an incorrect symbolic language in 20% and a faulty symbolic language in 4%. Pre-service teachers were observed to show the quadrilateral properties using substantially correct mathematical symbols. According to the findings, the pre-service teachers best expressed the properties of a square using the correct symbolic language, and the worst expression for the properties of a

trapezium. This might be due to the fact that pre-service teachers are more familiar with the properties of a square than the properties of a trapezium.

Table 6: Percentage and Frequency Values Related to Using the Verbal and Symbolic Language Together in the Context of Quadrilaterals

Language of Mathematics	Square		Rectangle		Parallelogram		Rhombus		Trapezium		Deltoid		Total	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
*Correct Verbal Expression-Correct Symbolic Representation	80	49	67	51	56	45	58	50	20	27	29	40	310	46
*Correct Verbal Expression-Incorrect Symbolic Representation	17	11	14	11	24	19	15	13	17	23	2	3	89	13
*Correct Verbal Expression-Missing Symbolic Representation	36	22	23	17	12	10	17	15	7	9	5	7	100	15
*Incorrect Verbal Expression-Correct Symbolic Representation	4	3	4	3	8	6	4	3	7	9	5	7	32	5
*Missing Verbal Expression-Correct Symbolic Representation	20	12	17	13	20	16	17	15	15	21	23	32	112	15
*Incorrect Verbal Expression-Incorrect Symbolic Representation	5	3	7	5	5	2	5	4	8	11	8	11	38	6
Total	162		132		125		116		74		72		681	

Table 6 shows the interpretation findings related to using verbal and symbolic language together in the context of quadrilaterals. According to the table, the pre-service teachers expressed 46% of the properties with correct verbal language and represented them using a correct symbolic language. They expressed 13% of the properties with a correct verbal language and with an incorrect symbolic language. They represented 15% of the properties symbolically correctly but failed to express them verbally. They expressed 6% of the properties both verbally incorrectly and symbolically incorrectly. According to the findings, less than half of the pre-service teachers could use verbal and symbolic language correctly to express the properties of quadrilaterals. The majority failed to do so. Pre-service teachers could express the properties verbally correctly, but they could not represent them symbolically or they misrepresented them. Although they represented the properties symbolically, they could not explain them verbally or they misexplained them. The sum of these categories equals to 48% and the majority of the pre-service teachers were only successful in using one of the two languages. This case can be attributed to the fact that the pre-service teachers have difficulty switching between verbal and symbolic languages. Their skills of using language of mathematics can be interpreted as poor.

Findings and Interpretations of the Quadrilateral Test and the Language Scale in Mathematics Teaching

Table 7. Descriptive Statistics Regarding the Pre-service Teachers' Scores of the Quadrilateral Test and the "Language in Mathematics Teaching Scale"

	N	Average	Ss	The Lowest Score	The Highest Score
"Language in Mathematics Teaching Scale"		73.70	8.30	52	90
Test related to Quadrilaterals	50	29.06	11.53	7	53
Square	50	7.1	3.49	1	16
Reactangle	50	6.2	2.60	0	11
Parallelogram	50	5.06	2.76	0	11
Rhombus	50	5.02	2.62	0	14
Trapezium	50	2.54	1.70	0	7
Deltoid	50	3.08	2.39	0	8

As shown in Table 7, the arithmetic mean score obtained from the "Language in Mathematics Teaching Scale" was found to be 73.70 and the standard deviation value was found to be 8.30. The average of the data obtained, which is 61.2-75.6, fall within the range of "I agree". It was concluded that pre-service teachers have positive views regarding the use of language of mathematics in mathematics teaching. Analyzing the scores of the test which was carried out to identify the skills of using the language of mathematics in the context of quadrilaterals, it was observed that the arithmetic mean score was 29.06, and the standard deviation value was 11.53. the average of the data was found to fall within the range of 30 and 18. It was concluded that the pre-service teachers used either of the verbal and the symbolic languages correctly and the other one incorrectly. These data support the finding that the pre-service teachers have difficulty using the verbal and symbolic language together. In addition to this, pre-service teachers were observed that they were most successful in explaining the properties of a square using the language of mathematics and least successful in explaining the properties of a trapezium using the language of mathematics. The pre-service teachers failed to effectively use the language of mathematics but had positive views of using the language in mathematics teaching.

CONCLUSION AND DISCUSSION

According to the research findings, half the pre-service mathematics teachers were successful in defining the types of quadrilaterals. The majority of the pre-service teachers did not mention some basic concepts while defining the types of quadrilaterals, so they made faulty definitions or misused the language of mathematics. The research studies regarding quadrilaterals in the literature also show that students have difficulty defining geometric shapes correctly (De Villiers, 1994; Erez & Yerushalmy, 2006; Fujita & Jones, 2007; Okazaki & Fujita, 2007). The pre-service teachers' responses show that they include such concepts like "tetragonal area" and "geometric object" in their definitions, which means they have inadequate information about quadrilateral types, they express mathematical concepts using inappropriate terms and they confuse the concepts. The reason why the pre-service teachers could not make accurate definitions might be attributed to their insufficient conceptual information about geometric shapes (Linchevsky, Vinner & Karsenty, 1992). Bozkurt and Koç (2012) also suggest that the pre-service teachers fail to define geometric concepts and confuse geometric shapes and geometric objects. Besides, the research studies conducted at different levels of education in Turkey prove that students are poor in terms of conceptual knowledge and geometry learning (Toluk, olkun & Durmuş, 2002; Ergün, 2010; Aktaş & Aktaş, 2011; Aktaş & Aktaş 2012; Türnüklü et al., 2013). The study concludes that a considerable number of the pre-service teachers fail to draw the shapes of quadrilateral types and show by mathematical symbols the basic properties on the drawings. The pre-service teachers have difficulty displaying the notations on the drawings and building a connection between shapes and the language of mathematics. However, geometry requires the appropriate use of language of mathematics in all dimensions of shape, definition and properties. The faulty drawings by the pre-service teachers might be due to the fact that the pre-service teachers do not feel the need to represent some properties such as "all sides are parallel and of equal length", and "angles are equal" rather than the fact that they do not know such features of the geometric shapes (Erşen & Karakuş, 2013). It can also be concluded that the rate of faulty drawings is high

because pre-service teachers have a habit of using quadrilateral images which they are familiar with rather than displaying the basic properties on geometric shapes (Üstün & Ubuz, 2004; Fujita & Jones, 2007; Okazaki & Fujita, 2007; Aktaş & Aktaş, 2012; Erşen & Karakuş, 2013).

Pirie (1998) explains the concepts of verbal and symbolic language, which are indispensable aspects of the language of mathematics and commonly used in mathematics classes (Çakmak, 2013; Emre, Sağ, Güllük & Argün, 2010; Yeşildere, 2007) as follows: Verbal language means the way of expressing mathematics in a verbal or written form using specific concepts and grammar structure. Symbolic language, on the other hand, means the way of representing mathematics with signs, symbols, and terms. The responses related to using verbal and symbolic languages to explain the properties of quadrilaterals indicate that a considerable number of the pre-service teachers explained the properties of quadrilaterals using verbal and symbolic language properly with the help of appropriate mathematical terms and notations. Although there was not a big difference between the rates of properly expressing the properties with a verbal language and representing the properties with a symbolic language, the difference was in favor of the verbal language. Capraro and Joffrion (2006) state that students experience challenges expressing mathematical information symbolically. The responses related to using verbal and symbolic languages together to express the properties of quadrilaterals proved that the pre-service teachers could not properly use verbal and symbolic languages together. What needs attention here is that the pre-service teachers were successful in using verbal and symbolic languages separately to express the different properties, whereas they failed to use both languages together. The pre-service teachers explained the properties of geometric shapes verbally; however, they failed to represent them symbolically and vice versa. This situation shows that they were either successful in using verbal or symbolic languages and they had difficulty understanding the connection between these two languages. Considering the skills of defining geometric shapes with appropriate mathematical concepts, drawing shapes and displaying mathematical notions on the shapes, using correct verbal and symbolic languages together to explain the features of a geometric shape, it can be concluded that the pre-service teachers have poor skills of using the language of mathematics. The research studies on the language of mathematics also indicate that students have problems expressing properly the mathematical information with the language of mathematics (Bozkurt & Koç, 2012; Dur, 2010; Gökkurt vd, 2013; Korhonen vd, 2011; Palabıyık & İspir, 2011; Rudd vd, 2008; Woods, 2009, Yeşildere, 2007, Yeşildere & Akkoç, 2010;). The inadequacies in the pre-service teachers' knowledge regarding the basic mathematical concepts are considered to act as a barrier to using the language of mathematics correctly (Yeşildere, 2007).

According to the findings, it can be noted that pre-service teachers have positive views of the necessity of using the language of mathematics in mathematics teaching. In addition to this, the pre-service teachers were observed to use either of the verbal or symbolic language properly while they used the other one incorrectly. They have difficulty using both languages together. Besides, it was found that although the pre-service teachers have poor skills of using the language of mathematics in the context of quadrilaterals, they have positive views of using the language in mathematics teaching.

The following are a number of suggestions for further research: Students should be given the opportunity to define the concepts in a verbal and written form, draw the shapes depending on the definitions, do activities using the verbal and symbolic languages together at all levels (from primary to university) in an effort to use the language of mathematics better and to overcome the challenges. Since teachers have a significant role in helping students improve their language of mathematics, they should pay attention to the ways of using verbal and symbolic languages that constitute the language of mathematics and enable the students to notice the transition and connection between these languages (Çakmak et al, 2014). Therefore, further studies should be conducted to explore the pre-service teachers' skills of using the language of mathematics. Besides, similar studies should be undertaken with a different sample and various possible variables.

IJONTE's Note: This article was presented at World Conference on Educational and Instructional Studies-WCEIS, 05- 07 November, 2015 by IJONTE Scientific Committee.

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REFERENCES

- Aktaş C. , M. & Aktaş, D.Y. (2011). 8. Sınıf öğrencilerinin dörtgenleri köşegen özelliklerinden yararlanarak tanıma sürecinin incelenmesi, 10. Matematik Sempozyumunda sözlü olarak sunulmuştur. İstanbul, Işık Üniversitesi.
- Aktaş, D.Y. & Aktaş C., M. (2012). 8. Sınıf Öğrencilerinin Özel Dörtgenleri Tanıma ve Aralarındaki Hiyerarşik Sınıflamayı Anlama Durumları. *İlköğretim Online*, 11(3), 714-728.
- Aydın, S. & Yeşilyurt, M. (2007). Matematik öğretiminde kullanılan dile ilişkin öğrenci görüşleri. *Elektronik Sosyal Bilimler Dergisi*, 6(22), 90-100.
- Bali-Çalıköğü, G. (2002). Matematik öğretiminde dil ölçeği. *Hacettepe Üniversitesi Eğitim Fakültesi dergisi*, 23, 57-61.
- Bali-Çalıköğü, G. (2003). Matematik öğretmen adaylarının matematik öğretiminde dile ilişkin görüşleri. *Hacettepe Üniversitesi Eğitim Fakültesi dergisi*, 25, 19-25.
- Barwell R., Leung C., Morgan C. & Street B. (2005). (eds). Special issue: language and maths. *Language and Education*, 19.
- Başaran, Ş. E. (1998). *Eğitim psikolojisi*. Ankara: Gül Yayınevi.
- Baykul, Y. (2000). *İlköğretimde Matematik Öğretimi*,4. Baskı, Pegem Yayıncılık Ankara.

Bozkurt, A. & Koç, Y. (2012). İlköğretim Matematik Öğretmenliği Birinci Sınıf Öğrencilerinin Prizma Kavramına Dair Bilgilerinin İncelenmesi, *Kuram ve Uygulamada Eğitim Bilimleri; Educational Sciences: Theory & Practice*, 12(4), 2941-2952.

Buchanan, T. (2007). The importance of teaching students how to read to comprehend mathematical language. Action Research Projects. Paper 5. <http://digitalcommons.unl.edu/mathmidactionresearch/5>.

Capraro, M. M. & Joffrion, H. (2006). Algebraic equations: can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, 27 (2), 147-164.

Chard, David. *Vocabulary Strategies for the Mathematics Classroom*. Boston, MA: Houghton Mifflin, 2003.

Çakmak, Z. (2013). Sekizinci sınıf öğrencilerinin istatistik konusundaki matematiksel dil becerilerine ilişkin değişkenlerin yapısal eşitlik modeli ile incelenmesi, Yayınlanmamış Yüksek Lisans Tezi, Erzincan Üniversitesi Fen Bilimleri Enstitüsü, Erzincan.

Çakmak, Bekdemir & Baş (2014). İlköğretim matematik öğretmenliği öğrencilerinin örüntüler konusundaki matematiksel dil becerileri. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi*, 16 (1), 204-223.

Çakmak, Konyalıoğlu & Işık (2014). İlköğretim Matematik Öğretmen Adaylarının Üç Boyutlu Cisimlere İlişkin Konu Alan Bilgilerinin İncelenmesi. *Middle Eastern & African Journal of Educational Research*, 8, 28-44.

Çetin, Ö. F. & Dane, A. (2004). Sınıf Öğretmenliği III. Sınıf Öğrencilerinin Geometrik Bilgilere Erişi Düzeyleri Üzerine, *Kastamonu Eğitim Dergisi*, 12(2), 427-436.

Dane, A. (2008). İlköğretim Matematik Öğretmenliği Programı Öğrencilerinin Nokta, Doğru ve Düzlem Kavramlarını Algıları. *Erzincan Eğitim Fakültesi Dergisi*, 10(2), 41-58.

De Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *Learning of Mathematics*, 14(1), 11-18.

Doğan M. & Güner, P. (2012). İlköğretim matematik öğretmen adaylarının matematik dilini anlama ve kullanma becerilerinin incelenmesi. *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi*, Niğde Üniversitesi, Niğde.

Dur, Z. (2010). Öğrencilerin matematiksel dili hikâye yazma yoluyla iletişimde kullanabilme becerilerinin farklı değişkenlere göre incelenmesi, Yayınlanmamış Doktora Tezi, Hacettepe Üniversitesi, Sosyal Bilimler Enstitüsü, Ankara.

Emre, E., Sağ, Y.G., Güllük, H. & Argün, Z. (2010, Eylül). Matematik öğretmen adaylarının matematiksel dil kullanımları. Çalışma 9. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresinde sunulmuş bildiri. Dokuz Eylül Üniversitesi, Buca Eğitim Fakültesi, İzmir.

Erez, M. & Yerushalmy, M. (2006). If you can turn a rectangle into a square, you can turn a square into a rectangle: Young students' experience the dragging tool. *International Journal of Computers for Mathematical Learning*, 11(3), 271-299.

Ergün, S. (2010). İlköğretim 7. sınıf öğrencilerinin çokgenleri algılama, tanımlama ve sınıflama biçimleri. Yayınlanmamış yüksek lisans tezi. Dokuz Eylül Üniversitesi, İzmir.

Erşen & Karakuş (2013). Sınıf öğretmeni adaylarının dörtgenlere yönelik kavram imajlarının değerlendirilmesi. *Turkish Journal of Computer and Mathematics Education*, 4(2), 124-146.

Ferrari-Luigi, P. (2004). Mathematical language and advanced mathematics learning. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (pp. 383-390).

Forman E. & van Oers B. (eds). (1998). Mathematics learning in sociocultural contexts. *Learning and Instruction*, 8: 469-72.

Fujita, T. & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing, *Research in Mathematics Education*, 9 (1-2), 3-20.

Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon. *The Journal of Mathematical Behavior*, 31, 60-72.

Glaserfeld, E. von (1995). *Radical Constructivism: A Way of Knowing and Learning*, London: Routledge/Falmer.

Gür, H. (Ed.). (2006). *Matematik öğretimi* (1. baskı). İstanbul: Lisans Yayıncılık.

Goldenberg, E. P., Cuoco, A. A. & Mark, J. (1998). A role for geometry in general education. In R. Lehrer & D. Chazan (Eds.), *Designing Learning Environments for Developing Understanding of Geometry and Space*, (pp. 3-44), Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Gökkurt, soylu & Gökkurt (2012). Öğrencilerin Matematik Öğretiminde Kullanılan Dile Yönelik Görüşlerinin Karşılaştırılması. *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi*, Niğde Üniversitesi, Niğde.

Gökkurt, Soylu & Örnek (2013). Mathematical language skills of mathematics teachers. *International Journal Of Academic Research*, 5(6), 238-245.

Haylock D. & Thangata, F. (2007). Key concepts in teaching primary mathematics. London: SAGE Publications Ltd., doi: <http://dx.doi.org/10.4135/9781446214503>

Hoyles C. & Forman E. (eds). (1995). Special issue: Processes and products of collaborative problem solving: some interdisciplinary perspectives. *Cognition and Instruction*, 13.

Korhonen, J., Linnanmäki, K. and Aunio, P. (2011). Language and mathematical performance: a comparison of lower secondary school students with different level of mathematical skills, *Scandinavian Journal of Educational Research*, 1-12.

Kuzniak, A. & Rauscher, J.C. (2007). On the geometrical thinking of pre-service school teachers. Proceedings Cerme4, Sant Feliu de Guixols Spain.

Larson C. (2007). The importance of vocabulary instruction in everyday mathematics. Scottsbluff, Nebraska. In partial fulfillment of the MAT degree Department of Mathematics University of Nebraska-Lincoln. Retrieved from scimath.unl.edu/MIM/files/research/LarsonC.pdf

Linchevsky, L., Vinner, S. & Karsenty, R. (1992). To be or not to be minimal? Student teachers views about definitions in geometry. In W. Geeslin & K. Graham (Eds.), *Proceedings of the sixteenth international conference for the psychology of mathematics education*, Vol. 2 (pp. 48-55). Durham USA.

Meaney, T. (2005) 'Mathematics as text', in Chonaki, A. and Christiansen, I. M., eds., *Challenging Perspectives on Mathematics Classroom Communication*, Westport, CT: Information Age Publishing, pp.109-141.

Monaghan F. (1999). Judging a word by the company its keeps: the use of concordancing software to explore aspects of the mathematics of register. *Language and Education* 13: 59–70.

Moschkovich, J. (2012). Mathematics, the Common Core and language: Recommendations for mathematics instruction for ELs aligned with the Common Core. Paper presented at the Understanding Language Conference at Stanford University, Stanford, CA.

National Council of Teachers of Mathematics (NCTM; 2000) Principles and Standards for School Mathematics <http://www.nctm.org/standards/content.aspx?id=16909s> (27.10.2015).

Ní Ríordáin, M. (2009). The role of language in teaching and learning mathematics. *Resource & Research Guides*, 1(1), 1-4.

Okazaki, M. & Fujita, T. (2007) . Prototype phenomena and common cognitive paths in the understanding of the inclusion relations between quadrilaterals in Japan and Scotland. In J. Woo, H. Lew, K. Park & D. Seo (Eds.). Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education (Vol:4, pp. 41-48).

Orton, A. & Frobisher, L. (1996). *Insights into teaching mathematics*. London:Cassell.

Otterburn, M. K. & Nicholson, A. R. (1976). The language of mathematics. *Mathematics in School*, 5(5), 18-20.

Palabıyık, U. & İspir, O. A. (2011). Örüntü Temelli Cebir Öğretiminin Öğrencilerin Cebirsel Düşünme Becerileri ve Matematiğe Karşı Tutumlarına Etkisi. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi*, 30 (2), 111-123.

Patton, M. Q. (2002). Variety in qualitative inquiry: Theoretical orientations. In C. D. Laughton, V. Novak, D. E. Axelsen, K. Journey, & K. Peterson (Eds.), *Qualitative research & evaluation methods* (pp. 132-133). London: Thousand Oaks.

Pickreign, J. (2007). Rectangle and Rhombi: How well do pre-service teachers know them? IUMPST, 1, 1-7. Retrieved January 22, 2013, from <http://files.eric.ed.gov/fulltext/EJ835492.pdf>

Pimm, D. (1987). *Speaking mathematically: communication in mathematics classrooms*. Routledge & K. Paul. London.

Pirie, S. E. B. (1998). Crossing the gulf between thought and symbol: Language as stepping-stones. In H. Steinbring, M. G. B. Bussi and A. Sierpiska (Eds.), *Language and Communication In The Mathematics Classroom* (pp.7-29). Reston, NCTM Publication.

Pusey, E.L. (2003). *The Van Hiele model of reasoning in geometry: a literature review*. Mathematics Education Raleigh. North Carolina State University.

Raiker, A. (2002). Spoken Language and mathematics. *Cambridge Journal of Education*, 32 (1), 45-60.

Reys, R., Suydam, M., & Lindquist, M. N. (1995). *Helping children learning mathematics*. Boston, MA: Allyn & Bacon.

Rudd, L. C., Lambert, M. C., Satterwhite, M. & Zaier, A. (2008). Mathematical language in early childhood settings: What really counts?, *Early Childhood Education*, 36, 75-80.

Sandt, S. & Nieuwoudt, H., D. (2003). Grade 7 teachers' and prospective teachers' content knowledge of geometry. *South African Journal of Education*.23(3), 199-205.

Sfard A. (2000). Symbolizing mathematical reality into being: How mathematical discourse and mathematical objects create each other. In P. Cobb, K.E. Yackel and K. McClain (eds). *Symbolizing and communicating: perspectives on mathematical discourse, tools, and instructional design*. Mahwah, NJ: Erlbaum, pp 37-98.

Sfard A. & Kieran C. (2001). Cognition as communication: Rethinking learning-by-talking through multifaceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, 8: 42-76.

Sherard, W. H. (1981). *Why is Geometry a Basic Skill?*. Mathematics Teacher.

Toluk, Z., Olkun, S. & Durmuş, S. (2002). Problem merkezli ve görsel modellerle destekli geometri öğretiminin sınıf öğretmenliği öğrencilerinin geometrik düşünme düzeylerinin gelişimine etkisi. *Beşinci Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi*, 16-18 Eylül, Ankara.

Toptaş, V. (2015). Matematiksel dile genel bir bakış. *International Journal of New Trends in Arts, Sports&ScienceEducation*, 4(1), 18-22.

Türnüklü, E., Gündoğdu-Alaylı, F. & Akkaş, E. N. (2013). Investigation of prospective primary mathematics teachers' perceptions and images for quadrilaterals. *Educational Sciences: Theory & Practice*, 13(2), 1225-1232.

Üstün, I. & Ubuz, B. (2004). Geometrik kavramların Geometer's Sketchpad yazılımı ile geliştirilmesi. *Eğitimde İyi Örnekler Konferansı*, 17 Ocak, İstanbul.

Van Hiele, P.M. (1986). *Structure and Insight. A Theory of Mathematics Education*. Orlando, Florida. Academic Press USA.

Woods, G. (2009). An investigation into the relationship between the understanding and use of mathematical language and achievement in mathematics at the foundation stage, *Procedia Social and Behavioral Sciences*, 1, 2191-2196.

Yeşildere, S. (2007). İlköğretim matematik öğretmen adaylarının matematiksel alan dilini kullanma yeterlikleri, *Boğaziçi Üniversitesi Eğitim Dergisi*, 24(2), 61-70.

Yeşildere, S. & Akkoç, H. (2010). Matematik öğretmen adaylarının sayı örüntülerine ilişkin pedagojik alan bilgilerinin konuya özel stratejiler bağlamında incelenmesi. *Ondokuz Mayıs Üniversitesi Eğitim Fakültesi Dergisi*, 29(1), 125-149.