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ANALYSIS OF MATHEMATICS TEACHER CANDIDATES' CONCEPTUAL KNOWLEDGE RELATED TO SEQUENCES

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Abstract
The purpose of this study is to examine the conceptual knowledge of mathematics teacher candidates about the sequence concept. This research is a case study in which qualitative methods are adopted. The first phase of the study was conducted with a total of 45 teacher candidates taking the course of Analysis III. In this stage, the "Sequence Knowledge Test" consisting of four open-ended questions developed by the researchers was used as a data collection tool to examine the concepts of the teacher candidates about the sequence concept. In the second stage of the research, individual interviews were made with 8 teacher candidates selected from the sample in order to conduct an in-depth study on conceptual knowledge about the subject of the sequence. Content analysis was used to analyze the obtained data. As a result of the analysis of the data, important shortcomings were found in the concept knowledge of the teacher candidates.

Keywords: Mathematics education, conceptual knowledge, sequences.

INTRODUCTION

Concepts are the way in which objects or events that can be related to each other are separated into mental groups or categories. Concepts are at the center of ideas and some theoreticians think of concepts as "the smallest building blocks or units of thought" (Ormrod, 2006). The importance of concepts in the mathematics education process is well known. Hiebert and Carpenter (1992) likened the formation process of conceptual knowledge to a network structure and emphasized that conceptual knowledge can also be formed by this network structure. The formation of conceptual knowledge can be achieved through the internalization and association of mathematical knowledge (Hiebert & Lefevre, 1986). The level of conceptual knowledge is the level of knowing the concept of the student (Rittle-Johnson ve Koedinger, 2002; Star, 2002). According to Fennema and Franke (1992), a mathematics teacher with conceptual knowledge is able to understand the underlying concepts of operations, demonstrates the ability to perceive the relationships within concepts and is capable of establishing some relations between mathematical concepts and real life applications of concepts.

In our daily lives, making events, ideas, and objects into a sequence is the process of putting them in a logical sequence. In mathematics a sequence is the process of putting more mathematical concepts (polynomial, matrix, function etc.), especially "numbers". This process occurs in the form of indexing, such as the first term, the second term, ... Mathematically this process is looked at as a function and functions defined from the set of positive integers Z^+, to set of real integers R are called " sequence" (Argun, Arıkan, Bulut and Halıcıoğlu, 2014). The sequence is one of the most important building blocks of mathematical analysis. As can be seen from the above description, besides having the concept of the sequence, the concept of series is built on sequence, and the concept of integral is built on the concept of series. Because the sequence are special functions (Balcı, 2008), it is very important that the subject is understood especially by the students of the analysis course. The sequences is an invaluable opportunity for students with learning disabilities about their functions and

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features. "If the sequence is a special function, what is the function? Does the sequence provide the conditions for functioning? What does convergence and limit in the sequence mean? Why is the limit only for $n \to \infty$ in the sequence defined? Are the known $\varepsilon - \delta$-techniques for limits in functions used in sequence? If so, how? What is the accumulation point? Why do not we talk about continuity in the sequence?" such questions provide important opportunities for teaching in the classroom to reinvestigate and reinforce knowledge of mathematical analysis. Moreover, the sequence is a rare subject that is prone to a real life based teaching model, which can be explained with everyday examples in analysis lessons. Also the working logic of computer programming (Basic, Logo, etc.) and some programs (Microsoft-Excel, Access, etc.) overlap with the sequences; therefore sequences are a very suitable subject for computer-aided instruction. Taken all this into consideration, it is seen how important and indispensable the "sequences" is, especially for high school and university level mathematics learners. Mathematics teachers sometimes say that learners have a significant learning difficulty about the concept of sequences. Despite this, however, there are no studies in the literature that examine students' learning difficulties and conceptual knowledge related to the sequences. Only in some studies it is seen that the sequences and series are in the upper order in the order of the subjects which are perceived as difficult by the students (Durmuş, 2004; Gurbuz, Toprak, Yapıcı and Doğan, 2011; Tatar, Okur and Tuna, 2008; Tuna and Kacar, 2005; Kutluca and Baki, 2009). In addition to these, Doruk and Kaplan (2013) examined the proof evaluation skills of elementary mathematics teacher candidates on the concept of convergence of sequences and Çiltaş and Işık (2012) examined the mental models of primary and secondary mathematics teachers' sequences and series. In other studies, concept difficulties related to sequence and series convergence were investigated (Akgün and Duru, 2007; Alcock and Simpson, 2004 ve 2005). Therefore, there is no example of a study in which conceptual information about sequence is examined and there is no example how the concept of sequence is perceived by math learners. Considering this lack of field literature and mathematical significance of sequence, this study aims to examine the conceptual knowledge of teacher candidate about sequence and so this study is thought to contribute significantly to the field literature. For this purpose, in this study, probing answers were searched as "how do the teacher candidates describe the sequence?"

METHOD

Model of study

This study aimed at examining the conceptual knowledge of mathematics teacher candidates about sequence is a case study in which qualitative methods are adopted. In the case study, an in-depth study is conducted focusing on an event, a case, an individual, or groups (Fraenkel & Wallen, 2000; Yildirim and Simsek, 2005).

Sample

Two different sampling methods were used in the selection of participants in the study. The study was first carried out with a total of 45 teacher candidates who were studying in the 3rd grade of Mathematics Teaching at a university in the north of Turkey in the fall semester of 2016-2017 academic year and who took the course of Analysis-III in the sequences-series topics. In this sampling selection a simple random sampling method has been adopted (Gay, Mills, & Airasian, 2006). As they have already taken the Analysis I and Analysis II courses (second grade), it is assumed that the participants have the necessary background knowledge on the sequences topic. In the selection of the participants in the second sample, the sampling method was adopted from the purposeful sampling methods (Patton, 2002). At this stage, individual interviews were conducted with 8 teacher candidates who gave incorrect answers to the items included in the data collection tool among the participants. In this way, it is aimed to conduct an in-depth examination of the conceptual knowledge of teacher candidates.

Data collection tool

In this study, the "Sequence Knowledge Test" consisting of four open-ended questions developed by researchers was used as a data collection tool to examine conceptual knowledge about the sequence concept of teacher candidates. The suitability of the questions in the prepared data collection tool for the purpose of measurement and how much it represents the area to be measured, is determined according to the "expert opinion" (Karasar, 1995). Open-ended questions allow students to freely express their thoughts about the research topic, allow scientific ideas and concept knowledge of the students to be released (Bauner and Schoon, 1993). In addition, interviews were held with pupils who provided certain criteria among the teacher candidates who participated in the study in order to conduct an in-depth examination of conceptual knowledge on the subject of the sequences.

Analysis of Data

Content analysis was used to analyze the data obtained in the study. According to Patton (2002), content analysis allows a framing of the collected data, provides the concretization of this frame by coding and categorization. The answers given by the teacher candidates participating in the study to the items in the sequence knowledge test were examined in three categories; correct, incorrect and empty. The data were independently coded and analyzed by two researchers who were experts in mathematics education. The percentage of agreement between the researchers' coding according to the reliability study was 85% (Miles and Huberman, 1994). The incompatible items were re-examined and a consensus was reached. Descriptive statistical techniques (percentage / frequency) were used in the analysis of the data obtained from the relevant test. And the data obtained from the interviews carried out with the teacher candidates were interpreted by the phenomenological method. This method is particularly used in studies where learning differences and the causes of these differences are being investigated (Marton, 1994). Phenomenographic method focuses not on individual but on differences in how individuals understand concepts, how they understand and interpret them (Marton and Booth, 1997).

RESULTS

In this section, the questions in the " sequences knowledge test" included analysis findings of the answers given by prospective teachers and interviews with candidates who gave different answers. Some of the answers that are required due to the pattern of the research are presented as an example. The answers that the candidates gave to the questions in the Sequences Knowledge Test were categorized as "right", "wrong" and "empty" and the findings were evaluated according to the order of the questions.

The answers given to the items in the test to see how teacher candidates identify the sequences are analyzed in this section, and the findings are given in Table 1 and Table 2.

Table 1: Analysis of the Responses to the First two Items

Criteria	Questions		
	1(Definition)	1(Sample)	2
	f (%)	f (%)	f (%)
Right	23 (51)	16 (36)	14 (31)
Wrong	19 (42)	18 (40)	28 (62)
Empty	3 (7)	11 (24)	3 (7)

The answers to the item "What is the sequence? In short, describe and give a sample of the sequences from daily life.", which is the first part of the test used in the research, were analyzed in two parts. According to this, only 51% (n = 23) of the candidates gave the right, 19% (n = 42) gave the wrong definitions to the part of the item related to the definition of sequence and 3% (n = 7) left it empty. For the second part of the item, the proportion of those who could give suitable daily samples was reduced to 36% (n = 16), the rate of giving wrong samples was 40% (n = 18) and the nulls were 24% (n = 11). This indicates that candidates can not make the exact definition of the sequence in their minds and it shows that candidates can not give proper examples of daily life related to the sequence concept, which is a sorting process, and can not practice theoretical knowledge. For example, some examples of incorrect answers given by students in this question are given in Figure 1.

Dizi = Tanım kümesi Doğal sayılar olan ve bir kurala göre artan veya azalan sayılardır. TV dizileri buna örnektir.

Figure 1-a: "Sequences: Numbers defined in the set of natural numbers and increasing or decreasing according to a rule. TV-sequences can be an example."

The interview made with the candidate teacher who answered in Figure 1-a is given below.
Researcher: Could you explain why you answer this question like that?
Teacher candidate: TV-series is also increasing, just like the sequences defined in mathematics.
Researcher: You said in the description that you gave "definition set of numbers with natural numbers". What does it mean?
Teacher candidate: I think the "numbers" expression is the element of a function's image set.
Researcher: Should a sequence be necessarily increasing or decreasing?
Teacher candidate: Actually you are right. There were also constant sequences, they are neither increasing nor decreasing, are they?

Here, the teachers candidate argue that the sequences must be increasing or decreasing. Therefore, it can be said that the candidate has formed the concept of sequences with such a relation in mind. However, he hesitates when he is told something different, indicating that he can not be sure of the conceptual knowledge he has about the sequences.

1) Dizi, belirli bir düzen halinde ilerleyen topluluktur.
Apartmanlar

Figure 1b:"The sequence is a community that moves in a certain order. For example, apartments."

The interview with the student who answers in Figure 1-b is given below.
Researcher: Could you explain why you answer this question like that?
Teacher candidate: In the sequences there is always an order, a certain rule. That's why I answered that.
Researcher: Do we call everything "sequence" that has the rule?
Teacher candidate: The general term in the sequence is a rule.
Researcher: What is the rule in apartment buildings?
Teacher candidate: Each apartment has a number and these numbers go up?

It is seen that this teacher candidate is also associated with an increase in the sequence concept. Some features of the sequences (increasing, decreasing, constant) seem to lead to the definition of the concept.

When the answers to the second ranked item in the test "Draw a graph of the sequence $f(n) = n^2 - 4$" are examined, it was seen that only 31% (n = 14) of participants did correctly, 62% (n = 28) misdrawled and 7% did not draw. When the false answers given to this question were examined, it was seen that almost all of the participants had drawn a graph of a function defined in the R-real numbers.

Figure 2a: The interview with the student who answered the question in Figure 2-a is given below.

Researcher: What do you think about drawing the graph here? Can you briefly tell me?
Teacher candidate: $f(n) = n^2 - 4$ this is a parabola equation, the wings up and focus point is $(0, -4)$.
Researcher: What is the sequence? Could you give me a mathematical definition?
Teacher candidate: The sequences are functions whose definition set is positive integers.
Researcher: Is not there a mistake in your drawing according to your description?
Teacher candidate: Of course! There will not be a left side in the chart, right?

In the case of the above interview, it is understood that the teacher candidate sees the sequence as a function whose definition set is a set of real numbers. Moreover, despite the fact that the researcher draws attention to the fault, the fact that the candidate still can not answer correctly shows that the candidate does not correctly understand the concept of sequence in his mind.

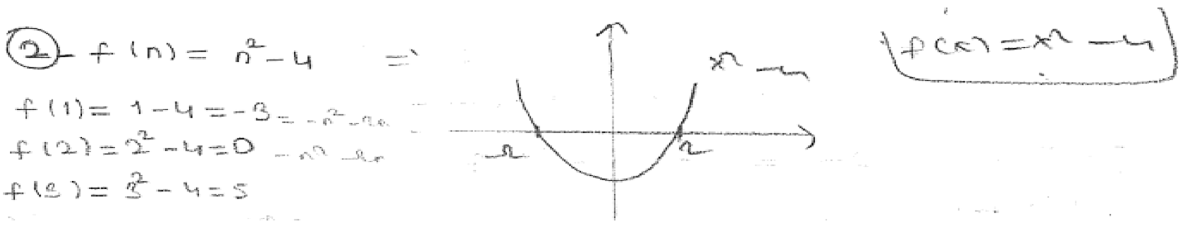

Figure 2b: The interview with the student who answers in Figure 2-b is given below.

Researcher: What do you think about drawing the graph here? Can you briefly tell me?
Teacher Candidate: I found the X-axis cut points, i thought of it as a parabola and i drew it.
Researcher: On the left side you find the values of function 1, 2 and 3, why did you do it?
Teacher Candidate: Because the sequence are defined in positive integers.
Researcher: Why do not you just mark these points in the coordinate system when you draw drawing?
Teacher Candidate: Because these points would not be enough for the parabola image.

Again, in this example, although the candidate has correctly given the mathematical definition of the sequence concept, it appears that it does not take this definition into consideration.

Table 2: Analysis Of The Answers To The Third And Fourth Questions

Criteria	Questions		
	3 (Arithmetic sequence)	3 (Geometrical sequence)	4
	f (%)	f (%)	f (%)
Right	19 (42)	19(42)	0 (0)
Wrong	15 (33)	16 (35)	43 (96)
Empty	11 (24)	10 (22)	2 (4)

The answers to the third item "Give some examples for arithmetic and geometric sequence from daily life" are analyzed in two parts. According to this, for arithmetic sequence examples only 42% of the

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candidates (n = 19) were right, 33% (n = 15) gave the wrong samples and 11% (n = 24) left the item empty. For the second part of the item, the rate of giving geometric sequential daily life samples is again 42% (n = 19), while the false sample is 35% (n = 16) and the nulls are 22% (n = 10).


3) Aritmetik dizi
Birinci rafta bulunan 2kitap
ile ikinci rafta bulunan 3kitap
Domino
taşları bir
aritmetik
Geometrik dizi
küplerin alanları bir
geometrik dizidir.

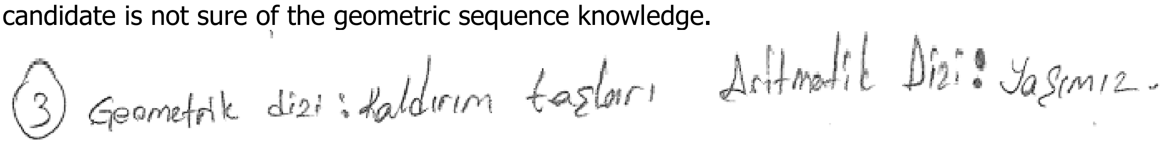
Figure 3a:"Arithmetic sequence: 2 books for the first shelf, 3 books for the second shelf and so on, domino stones form an arithmetic sequence.

Geometric sequence: The cubic field is a geometric sequence "

The interview with the student who answers in Figure 3-a is given below.

Researcher: Could you explain the example you gave for the arithmetic sequences?
Teacher Candidate: I wanted to give an example of books on the shelf for arithmetic sequences. 2 for the first shelf, 3 for the second shelf, and so on. Also in the dominoes, the numbers increase one by one.
Researcher: Could you explain the example you gave for the geometric sequences?
Teacher candidate: We saw in the lessons, nested cubes. These are geometric sequences.
Researcher: Where do the nested cubes form the geometric sequences?
Candidate Teacher: The areas of the cubes placed inside form the geometric sequence.

As you can see in the example above, the candidate is constantly associating the arithmetic sequences with an increase and it is also noteworthy that the examples given in increase especially one by one. Looking at the example given for the geometric sequence, it is understood that the candidate is not sure of the geometric sequence knowledge.


3 Geometrik dizi: Kaldırım taşları Aritmetik Dizi: Yaşımız.

Figüre 3b: "Geometric sequence: Pavement Stones Arithmetic sequence: Our age"

The interview with the student who answered as in Figure 3-b is given below.

Researcher: Could you explain the example you gave for the geometric sequence?
Candidate Teacher: Pavement stones have geometric shapes.
Researcher: Could you explain the example you gave for the arithmetic sequence?
Teacher candidate: In the case of age, only numbers are used.
Researcher: Do we use geometric sequence in the case of geometry?
Teacher candidate: Yes, because we need to do geometric calculations.

This example clearly demonstrates that the conceptual knowledge of mathematics learners should be examined. Here it appears that the candidate has identified concepts with their names. It is understood that the candidate sees the arithmetic sequence as a sequence where arithmetic operations can be performed, and the geometric sequence as a sequence containing the geometric order.

None of the answers to the last item in the form of "Find the general term of a sequence $(a_n) = \{1, 1/2, 1/3, 1/4, 1/5, \ldots\}$ " can not be judged correctly, almost all 96% (n=43) of the answers are wrong. The rate of those who leave this item is 4% (n = 2).

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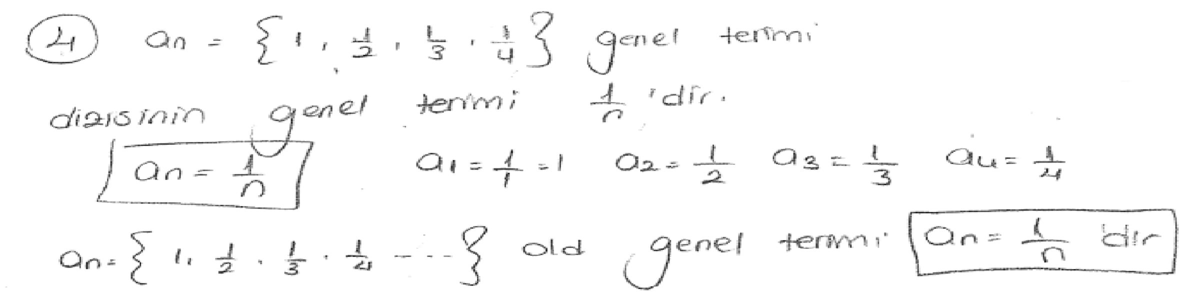

Figure 4a: The interview with the student who answered as in Figure 4-a is given below.

Researcher: From the first four terms, you have come to the general term. Can not the fifth term be based on a different rule?
Teacher Candidate: How would it be? It probably will not be different, i suppose the fifth term would be 1/5, I guess.

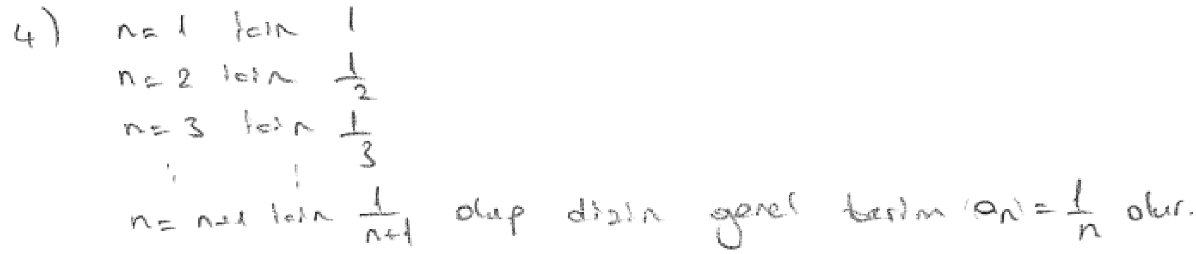

Figure 4b: The interview with the student who answers as in Figure 4-b is given below.

Researcher: From the first four terms, you have come to the general term. Can not the fifth term be based on a different rule?
Teacher Candidate: Could it be? I do not think so, but in such questions the first few terms are given and there is a general term. There is the same thing here.

As can be seen in the two examples given above, candidates think that the general term of the sequence can be found by knowing several terms of the sequence.

CONCLUSION AND DISCUSSION

In this study, the conceptual knowledge of teacher candidates about the sequence was examined. According to the findings of the research, it is possible to reach the following conclusions about the concept knowledge of the candidates on the sequence;
Candidates were able to give a mathematical description of the concept of the sequence (51%), but it was found that candidates were given difficulty (36%) to give daily examples of the sequence. It is seen that the majority of the teacher candidates who give the wrong conceptual definition, are associated the sequence with an increase or decrease. It can be said that this association is dominant in the examples given by teacher candidates about the sequence. Similar conclusions about the difficulties involved in the transfer of mathematical knowledge into daily life are evident in the work of Koirala and Bowman (2003), Guberman (2004) and Moschkovich (2007). The fact that the percentage of correctly drawn participants in the test is low (31%) gives important ideas about candidates' knowledge about the concept of the sequence. Candidates see the sequence as a continuous function defined in the set of real numbers and make their drawings accordingly. The drawings are usually meant to include a set of definitions $R^{-} \cup \{0\}$, and graphics are often curves, not dot clusters. This suggests that candidates do not bring the sequence definition into their minds. In fact, it has been shown in many studies that students generally have difficulties in graphical drawing (Capraro, Kulm and Caproro, 2005, Shah and Hoeffner, 2002, Uyanik, 2007).

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It has been seen that the teacher candidates generally regard the arithmetic sequence as consecutive number sequences, because in the examples it has been determined that the difference between successive terms is often one. It can be said that candidates often give examples of pattern samples for arithmetic sequences or sums of sequences, because of their misperception about the concept of arithmetic sequence.

When looking at the examples given for geometric sequences, it is seen that the samples often overlap with the sequences of samples solved in the courses such as "finding the way from the given height to the left of the dropped ball" or "finding the sum of the geometrical properties of the shapes that are nested and that are getting smaller or bigger in a certain rate ". Apart from these, the lack of different examples about geometric sequences shows that the teacher candidates have difficulty in transferring the information they have seen in the course to daily life. Studies show that teachers do not have a connection between the mathematical topics in university education and the topics they teach in their schools (Wu, 1999). Again, Soylu and Soylu (2006), in the study of students' problem solving processes, concluded that students could not apply the mathematical concepts or definitions they learned in the course. According to Galbraith and Stillman (1998), while secondary school students know mathematical formulas by heart, they do not understand the concepts, so they have difficulty using their knowledge to solve real-life problems. With the studies mentioned here, it is seen that these research results emphasize similar points. For this reason, one of the priorities of the instructors who teach mathematics teacher candidates in the education faculties should be to create environments that will enable the teacher candidates to understand the nature and basic characteristics of mathematical concepts. Moreover, it is striking that most of the samples given by the candidates are geometric shapes. This indicates that the candidates have attached meaning to the concepts by associating them with only their names.

It is quite striking that no candidate can respond correctly to the last item in the information test, " Find the general term for the $(a_n)=\{1,1/2,1/3,1/4,1/5,...\}$ " However, the general term of only a few terms known sequences, can not be determined. Here, the first five terms of both of the $(1/n)$ and $((n-1)(n-2)(n-3)(n-4)(n-5)+1/n)$ sequences are this form $\{1,1/2,1/3,1/4,1/5\}$. However, these are two different sequences, also there are other examples for sequences that have this first five terms (Balcı, 1996). However, the sixth terms of these two sequences are $1/6$ and $721/6$, respectively. This is an important aspect of sequences, but none of the candidates have unfortunately paid attention to it. It is understood that the candidates here are only trying to find the general term by focusing on the rule. Parallel to this situation, according to Sevimli (2009), argues that rule-based learning is more dominant in the analysis courses, and account-based approaches are adopted.

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